Algebra

$$(x+k)(x-k) = x^2 - k^2$$

$$(x+k)^2 = x^2 + 2kx + k^2, \quad (x-k)^2 = x^2 - 2kx + k^2$$

$$x^3 \pm k^3 = (x \pm k)(x^2 \mp kx + k^2)$$

Formula for solving a quadratic equation:

$$\text{if } ax^2 + bx + c = 0 \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Laws of Indices:

$$a^m a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n} \qquad (a^m)^n = a^{mn}$$

$$a^0 = 1 \qquad a^{-m} = \frac{1}{a^m} \qquad a^{1/n} = \sqrt[n]{a} \qquad a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

Laws of Logarithms:

For any positive base b (with $b \neq 1$)

$$\log_b A = c \quad \text{means} \quad A = b^c$$

$$\log_b A + \log_b B = \log_b AB, \qquad \log_b A - \log_b B = \log_b \frac{A}{B},$$

$$n \log_b A = \log_b A^n, \qquad \log_b 1 = 0, \qquad \log_b b = 1$$

Formula for change of base:

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Logarithms to base e, denoted $\log_e$ or alternatively $\ln$ are called *natural logarithms*. The letter e stands for the exponential constant which is approximately 2.718.

Useful Symbols and Notations

$a \mid b$	a divides b
$a \not\mid b$	a does not divide b
a mod b	remainder when a is divided by b
$\lfloor x \rfloor$	floor of x; the greatest integer less than or equal to x
$\lceil x \rceil$	ceiling of x; the smallest integer greater than or equal to x
$\gcd(a, b)$	greatest common divisor of a and b
$\mathrm{lcm}(a, b)$	least common multiple of a and b
$P \equiv Q$	P and Q are logically equivalent
$\sum_{i=1}^{n} a_i$	$= a_1 + a_2 + \ldots + a_{n-1} + a_n$
$\prod_{i=1}^{n} a_i$	$= a_1 \times a_2 \times \ldots \times a_{n-1} \times a_n$
$\sum_{i \in S} a_i$	the sum of the elements in the set $\{a_i : i \in S\}$
$\prod_{i \in S} a_i$	the product of the elements in the set $\{a_i : i \in S\}$

For example, if S is the set of odd integers between 0 and 10, and $a_i = i$ then $\sum_{i \in S} a_i = 1 + 3 + 5 + 7 + 9 = 25$ and $\prod_{i \in S} a_i = 1 \times 3 \times 5 \times 7 \times 9 = 945$.