Sets

Set builder notation

- N = {0,1,2,3,…}
- R = reals
- Z = {… -3,-2,-1,0,1,2,3,…}
- Z+ = {1,2,3,4,5,…}

$$N = \{x \mid x \geq 0\}$$

$$\{1,2,3,1,2,3\} = \{1,2,3\} = \{3,1,2\}$$

Set builder notation

$$S = \{x \mid P(x)\}$$

S contains all elements from U (universal set)
That make predicate P true

$$Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

Brace notation with ellipses (wee dots)

Set Membership

$$x \in S$$

x is in the set S (x is a member of S)

$$y \notin S$$

y is not in the set S (not a member)

Set operators

$$B = \{1,3,5,7\}$$
$$C = \{1,2,3,4\}$$

$$A = B \cup C$$
$$A = B \cap C$$
$$B \subset C$$
$$B \subseteq C$$

Set empty

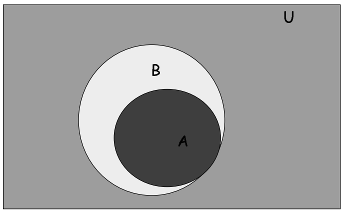
$$\emptyset$$

$$\{\ \}$$

The empty set

Venn Diagram

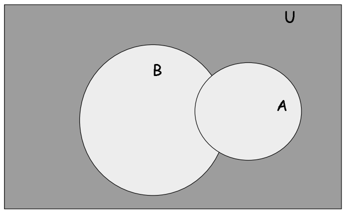
$A \subset B$



U is the universal set
A is a subset of B

Venn Diagram

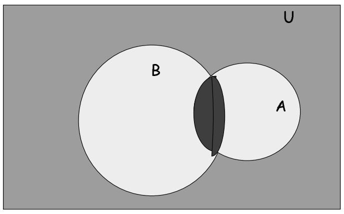
$A \cup B$



U is the universal set
A united with B

Venn Diagram

$A \cap B$



U is the universal set
A intersection B

Cardinality of a set

The number of elements in a set (the size of the set)

$S = \{1,2,3,7,9\}$
$|S| = 5$

Power Set

The set of all possible sets

$S = \{1,2,3\}$
$P(S) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$

Power Set

The set of all possible sets

$$|S| = n$$

$$|P(n)| = ?$$

Is this true for a set S?

$$S \subseteq S$$

$$\varnothing \subseteq S$$

Cartesian Product

$$A = \{1,2,3,4\}$$
$$B = \{a,b,c\}$$
$$A \times B = \{(1,a),(1,b),(1,c),(2,a),(2,b),(2,c),(3,a),(3,b),(3,c)\}$$

A set of *ordered* tuples

$$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$$

$$A \times B \neq B \times A$$

Subset

$$\forall x (x \in A \rightarrow x \in B) \Leftrightarrow A \subseteq B$$

- is empty {} a subset of anything?
- Is anything a subset of {}?
- We have an implication, what is its truth table?
- Note: improper subset!!

Try This

Using set builder notation describe the following sets

- odd integers in the range 1 to 9
- the integers 1,4,9,16,25
- even numbers in the range -8 to 8

Answers

Using set builder notation describe the following sets

- odd integers in the range 1 to 9
- the integers 1,4,9,16,25
- even numbers in the range -8 to 8

$$\{2x - 1 \mid 1 \leq x \leq 5\}$$

$$\{x^2 \mid 1 \leq x \leq 5\}$$

$$\{2x \mid -4 \leq x \leq 4\}$$

Go do this in claire

- build the sets we just mentioned
- test if {} is a subset of itself

Using set builder notation describe the following sets

- odd integers in the range 1 to 9
- the integers 1,4,9,16,25
- even numbers in the range -8 to 8

How might a computer represent a set?

Remember those bit operations?

Power set

Try this

Compute the power set of

- {1,2}
- {1,2,3}
- {{1},{2}}
- {}

Power set

Compute the power set of

- {1,2}
- {1,2,3}
- {{1},{2}}
- {}

Do it in claire

Think again … how might we represent sets?

Cartesian Product

$$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$$

A set of ordered tuples

- note AxB is not equal to BxA

Try This

Let A={1,2,3} and B={x,y}, find

- AxB
- BxA
- if |A|=n and |B|=m what is |AxB|

My answer

Do it in claire

Let A={1,2,3} and B={x,y}, find

- AxB
- BxA
- if |A|=n and |B|=m what is |AxB|

For the brave … the claire code

```
[member(e:any,A:set) : boolean
 -> exists(x in A | x = e)]
//
// There exists some element x in A such that
// x = e
//

[subset(A:set,B:set) : boolean
 -> forall(x in A | member(x,B))]
//
// All elements of A are in B. Sometimes called "improper" subset
// What does it do when A = {}? Hint: P -> Q and P is false!
//
```

```
[PS(A:set) : set -> PS(A,{})]

[PS(A:{{}},B:set) : set -> set(B)]

[PS(A:set,B:set) : void
 -> let x := A[1]
    in PS(delete(copy(A),x),add(copy(B),x)) U
       PS(delete(copy(A),x),B)]
//
// The power set of A
// NOTE: U is the (claire) union operator
//
```

```
[CP(A:set,B:set) : set
  -> let pairs := {}
     in (for x in A
          (for y in B pairs := add(pairs,list(x,y))),
         pairs)]
//
// The cartesian product of 2 sets A and B
// Produce a set of tuples (as a list)
//
// Demo: CP({1,2,3},{1,5})
//       CP({1,2,3},{"A","B"})
//
```