The Pigeon hole problem

If k+1 objects are placed in k containers then one container must contain at least 2 objects

Obviously!

The generalised pigeon hole principle

If n objects are placed in k containers, then at least one container has $\lceil n/k \rceil$ objects

Note: $\lceil x \rceil$ is the smallest integer greater than or equal to x

example

Given 100 people in a room, what is the minimum number of people that are born on the same month?

What are the pigeonholes?
What are the pigeons?
What do we know?
What can we think of?

$\lceil 100/12 \rceil = \lceil 8.333 \rceil = 9$

If n objects placed in k containers, then at least one container has $\lceil n/k \rceil$ objects

example

What is the minimum number of students required to be sure that at least 6 will receive the same grades, of A, B, C, D, and F (fail!)

5 containers: the marks A, B, C, D, F
At least 1 container has to have at least 6 students

- $\lceil students/5 \rceil = 6$
- $\therefore$ students = 26

If n objects placed in k containers, then at least one container has $\lceil n/k \rceil$ objects

More pigeon hole examples!

example

You are in a room with 10 people. ***There at least two people in the room who know the same number of people in the room.*** Is this true?

- What do we know … about knowing people?
- It's a pigeonhole problem …
  - So what are the pigeonholes?
  - What are the pigeons?

exercise

Let d be a positive integer greater than zero. Show that amongst d+1 (not necessarily consecutive integers) there are at least two with the same remainder when divided by d.

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- the pigeon holes are the remainders
  - there are at most d remainders
    - 0,1,2,…,d-1
- there are d+1 numbers
  - these are our pigeons
- since we have d+1 numbers and d remainders at least 2 have the same remainder

example

A bowl contains 10 red and 10 blue balls. A blindfolded woman selects balls at random, without replacement.

(a) How many balls must she select to be sure of getting at least 3 balls of the same colour?

- we have 2 colours, red and blue
  - these are our pigeon holes
- at least one pigeon hole must have at least 3 pigeons (coloured balls!)
- using the generalised pigeon hole principle
  - $\lceil n/k \rceil = 3$
  - where n is the smallest number of pigeons required
  - where k is the number of pigeon holes
  - $\lceil n/2 \rceil = 3$
  - $\therefore$ n = 5
- note, we were not constrained by the number of balls!

exercise

A bowl contains 10 red and 10 blue balls. A blindfolded woman selects balls at random, without replacement.

(b) How many balls must she select to be sure of getting at least 3 blue balls?

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(b) How many balls must she select to be sure of getting at least 3 blue balls?

- again, the colours correspond to our pigeon holes
  - we have 2 pigeonholes
- she could select 10 red balls one after the other
- so she would then need to select 3 more
- therefore she must select 13 to be sure of getting 3 blue balls!
- Note: in this case it did matter that we started with 10 of each colour

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