af2

Introduction

What's the course about?

- Discrete Mathematics
- The mathematics that underpins computer science
  - and other sciences as well.

What kind of things/problems will we cover?

- Logic
  - allows us to express ourselves concisely
  - allows us to reason about things (programs/structures/etc)
- Sets and Functions
  - sets: collections of objects, used for relations, counting, …
  - functions: mapping from one set to another
- Complexity
  - how long will it take a program to run?
- Methods of Proof
  - for argument's sake
  - being able to prove things
  - example, program correctness
- Counting
  how many 6 character passwords are there that have at least one digit, at least one lower case character and at least one upper case character
- Graphs and Trees
  - vertices (points) and edges (lines)
  - representing data structures, networks, the www, etc.
- Relations
  - data bases, constraints, …

Why bother with all of this?

To teach you
- mathematical reasoning and
- mathematical problem solving

How to learn discrete mathematics?

- Attend the lectures (obviously)
- work through the exercises in the course text book
- do the tutorial exercises

The more work you do yourself rather than passively reading or copying solutions, the more you will learn

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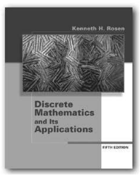
The course text book

Discrete Mathematics & its Applications
5th Edition
Kenneth H. Rosen

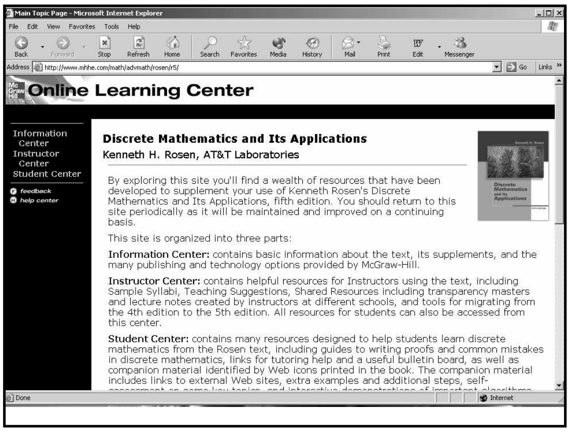


- You must buy this book
- The lectures follow this book
- There are no course notes
- The book is the course notes.

You will use the book in L2, L3, and L4, and beyond.

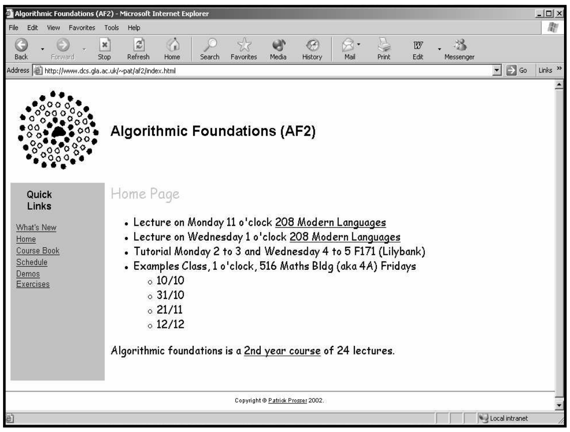
The web

- The book has its own web site
  - http://www.mhhe.com/rosen



The web

- af2 has a web site
  - http://www.dcs.gla.ac.uk/~pat/af2



What's on offer

- 23 lectures
- 22 drop in tutorials
- 2 assessed exercises
- 4 examples classes

22 lectures by Patrick Prosser

1st Lecture

The Foundations: Logic

Logic is the basis of all mathematical reasoning

The rules of logic give precise meaning to mathematical statements

Make sure your phone is off, please.

Propositions

- The basic building blocks of logic - propositions
- A declarative sentence that is true or false, but not both

Examples of Propositions:

- (a) Patrick Prosser is 21 years old
- (b) My dog has no nose.
- (c) 1 + 1 = 2
- (d) 2 + 2 = 3
- (e) Teddy eats fish

Propositions (c) and (e) are true [Teddy is one of Andrea's cats]

Propositions (a), (b), and (d) are obviously false

Propositions

The following are NOT Propositions:

- (a) What's on telly?
- (b) Drink tea.
- (c) x + 1 = 2
- (d) x + y = z

(a) and (b) are not declarative sentences, and are neither true or false
(c) and (d) have unassigned variables, and are neither true or false

Propositions

Truth value of a proposition is
- T for true, also 1 for true
  or
- F for false, also 0 for false

The area of logic that deals with propositions is called "*propositional logic*" or "*propositional calculus*"

Letters may be used to represent propositions, typically use the letters p, q, r, ...

Let p be the proposition "Today is Friday"
Let q be the proposition "It is raining"

Negation of a Proposition

Let p be the proposition "Today is Friday"
Let q be the proposition "It is raining"

¬p "It is not the case that today is Friday"
"It isn't Friday"
¬q "It is not the case that it is raining"
"It isn't raining"

"It is not the case that ..." sounds like we are speaking Vulcan. Avoid this

Negation of a Proposition

Truth table for negation

p	$\neg p$
F	T
T	F

Or equivalently

p	$\neg p$
0	1
1	0

Conjunction

Let p be the proposition "Today is Friday"
Let q be the proposition "It is raining"

p and q   or equivalently $p \wedge q$

$p \wedge q$
"It's Friday and it's raining"

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

Note: 1st two columns count up in binary as we move down the rows

Disjunction

Let p be the proposition "Today is Friday"
Let q be the proposition "It is raining"

p or q   or equivalently $p \vee q$

$p \vee q$
"It's Friday or it's raining"

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

Note: 1st two columns count up in binary as we move down the rows

Exclusive or

Let p be the proposition "Today is Friday"
Let q be the proposition "It is raining"

p xor q   or equivalently $p \oplus q$

$p \oplus q$
"It's either Friday or it's raining, but it's not both!"

p	q	$p \oplus q$
0	0	0
0	1	1
1	0	1
1	1	0

Note: 1st two columns count up in binary as we move down the rows

Exclusive or

In Big Tony's restaurant, for a starter it says "Soup or salad"

What does Tony mean?

(a) You can have no soup and no salad, no soup and salad, soup and no salad, or soup and salad
(b) You can have eith
(c) You can either
you can skip the sta

Hey Tony, was that an exclusive or?

English is typically imprecise, ... and I wouldn't get smart with Big Tony

Implication

Let p be the proposition "Today is Friday"
Let q be the proposition "It is raining"

p implies q   or equivalently $p \rightarrow q$

$p \rightarrow q$
"If it's Friday then it's raining

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Implication

Implication is often misunderstood

p implies q
if p then q
if p, q
q whenever p
…
see Rosen page 6

$p \rightarrow q$

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

$p \rightarrow q$
Think of this as a contract!

Implication

$p \rightarrow q$
Think of this as a contract!
The contract holds, or it doesn't

If it is sunny my Mum will take me to the beach

p: it is sunny
q: Mum will take me to the beach

p	q	$p \rightarrow q$	what does this mean?
0	0	1	It wasn't sunny and we didn't go to the beach. Good!
0	1	1	It wasn't sunny and we did go to the beach. That's okay.
1	0	0	It was sunny and Mum didn't take me to the beach. Bad Mum!
1	1	1	It was sunny and we went to the beach. Great!

Implication

$p \rightarrow q$

It's true with the one exception,
when p is true and q is false
i.e. when the contract is broken!

Implication

$p \rightarrow q$

It's true with the one exception,
when p is true and q is false
i.e. when the contract is broken!

P	Q	$P \rightarrow Q$
0	0	1
0	1	1
1	0	0
1	1	1

Don't let it be false $\neg (P \wedge \neg Q)$

When is it true? $\neg P \vee Q$

$$P \rightarrow Q \Leftrightarrow \neg P \vee Q$$

Converse, Contrapositive, and Inverse (Rosen page 8)

The converse of $p \rightarrow q$ is $q \rightarrow p$

The contrapositive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$

The inverse of $p \rightarrow q$ is $\neg p \rightarrow \neg q$

Converse

The converse of $p \rightarrow q$ is $q \rightarrow p$

p	q	$p \rightarrow q$	$q \rightarrow p$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

Converse

The contrapositive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$

p	q	$p \rightarrow q$	$\neg p$	$\neg q$	$\neg q \rightarrow \neg p$
0	0	1	1	1	1
0	1	1	1	0	1
1	0	0	0	1	0
1	1	1	0	0	1

NOTE: $p \rightarrow q$ is logically equivalent to it's contrapositive $\neg q \rightarrow \neg p$

Contrapositive

If it is sunny my Mum will take me to the beach

p: it is sunny
q: Mum will take me to the beach

$p \rightarrow q$

Contrapositive:
If Mum doesn't take me to the beach then it is not sunny

$\neg q \rightarrow \neg p$

biconditional

$P \leftrightarrow Q$

How we might say it:
- P iff Q
- P if and only if Q

$P \leftrightarrow Q$ is true when
- P is true and Q is true
- P is false and Q is false

Logical equivalence

$P \Leftrightarrow Q$

How we might say it:
- P is logically equivalent to Q

$P \Leftrightarrow Q$
may prove this
- by laws of logical equivalence or
- by a truth table or
- by some other line of reasoning

Biconditional (again!)

$P \leftrightarrow Q$

$P \leftrightarrow Q \Leftrightarrow P \rightarrow Q \wedge Q \rightarrow P$

Tautology

T

Something that is always true

Important stuff in this lecture

Know and understand implication (think of it as a contract)
It is central to understanding methods of proof

Know and understand contrapositive.
It is central to understanding methods of proof, in particular, converting a direct proof to an indirect proof (more later)

Convince yourself that implication and its contrapositive are equivalent

A reality check

Implication is often misunderstood

We are given 4 cards. Cards have a letter on one side and a number on the other side. We have the following rule

If a card has the number 3 on one side then it has the letter B on the other side

Given the 4 cards below, exactly what cards must be turned over to confirm that the above rule holds

B 7 C 3

A reality check

Implication is often misunderstood

p: card has the number 3 on one side
q: card has a B on the other side
p →q

B 7 C 3

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

Do we need to turn over this (red) card?

A reality check

Implication is often misunderstood

p: card has the number 3 on one side
q: card has a B on the other side
p →q

B 7 C 3

p	q	$p \to q$
0	0	1
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0	0	1
0	1	1
1	0	0
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Do we need to turn over this (red) card?

A reality check

Implication is often misunderstood

p: card has the number 3 on one side
q: card has a B on the other side
p →q

B | 7 | C | 3

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

We only need to turn over the purple cards.
Can you see why?

Resit question, 2002

More Examples

I eat olives when I am in Spain

If I am in Spain I eat olives

P: I am in Spain
Q: I eat olives

$P \to Q$

P	Q	$P \to Q$	*meaning*
0	0	1	*Not in Spain, not eating olive (okay with me)*
0	1	1	*Eating olives, not in Spain (no problem)*
1	0	0	*In Spain, not eating olives (unhappy)*
1	1	1	*In Spain, eating olives (happy)*

What is:
1. Contrapositive?
2. Inverse?
3. Biconditional?
4. Converse?
of the above?

$P \to Q$	*implication*
$Q \to P$	*converse of* $P \to Q$
$\neg Q \to \neg P$	*contrapositive of* $P \to Q$
$\neg P \to \neg Q$	*inverse of* $P \to Q$

More Examples

The car doesn't start whenever the battery is dead

P: the battery is dead
Q: the car doesn't start

P	Q	$P \to Q$	*meaning*
0	0	1	*Battery okay, car starts*
0	1	1	*Battery okay, car will not start (out of petrol?)*
1	0	0	*Battery is dead, car starts (yikes! Is this magic?)*
1	1	1	*Battery dead, car not starting (as advertised)*

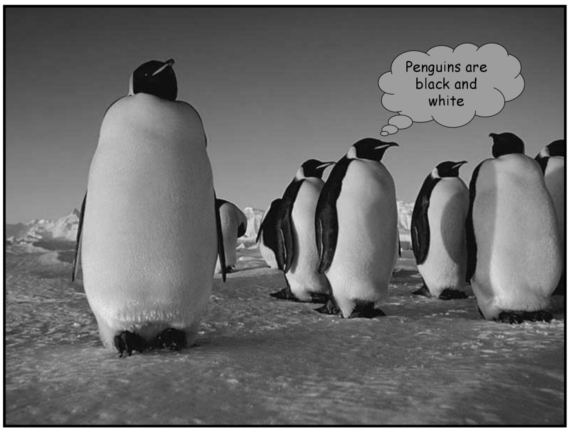
More Examples

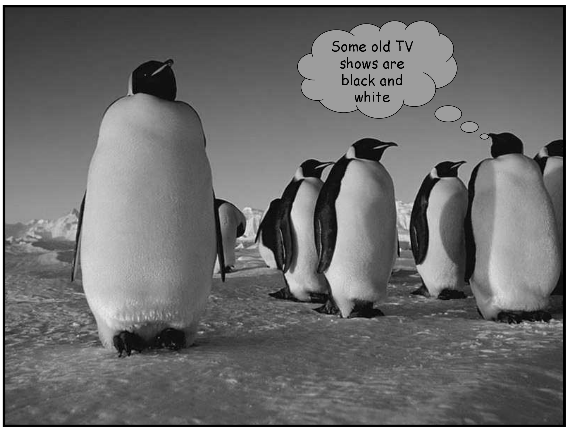
If the butler shot the Major he will have the gun in his hand.

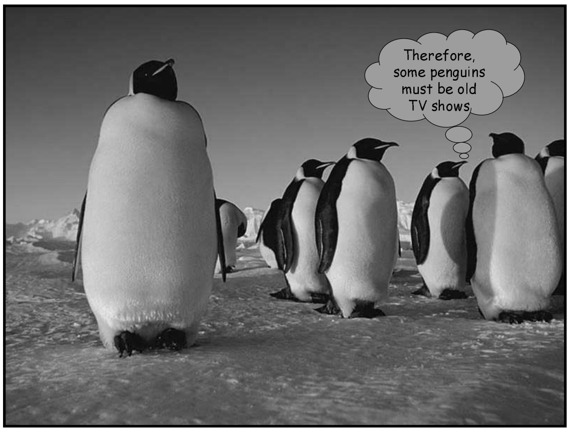
P: the butler shot the Major
Q: he has the gun in his hand.

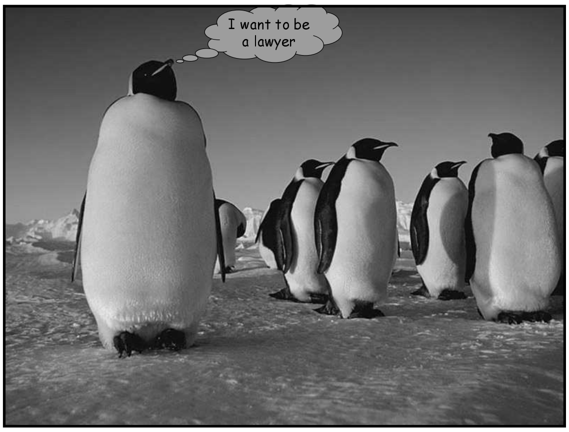
P	Q	$P \to Q$	*meaning*
0	0	1	*Butler did not shoot Major, does not have gun in hand (okay, innocent)*
0	1	1	*Butler did not shoot Major, does have gun in hand (okay, but what does jury think?)*
1	0	0	*Butler did shoot Major but does not have gun in hand (yikes! How did he do that?)*
1	1	1	*Butler shot Major, has smoking gun in hand (guilty!)*

Can you see how evidence can be misused?









Last example for now

This being, that becomes.
From the arising of this, that arises.
This not becoming, that does not become.
From the ceasing of this, that ceases.

P: this

Q: that

$$P \to Q \wedge \neg P \to \neg Q$$

P	Q	$P \to Q$	$\neg P \to \neg Q$	$P \to Q \wedge \neg P \to \neg Q$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	1	1	1

What is this?