Rules of Inference

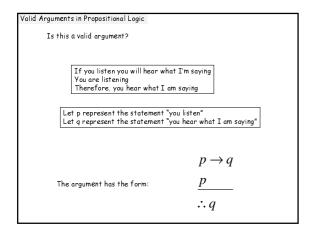
Rosen 1.5

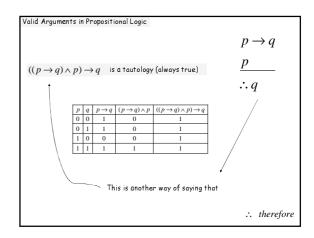
Proofs in mathematics are valid arguments

An argument is a sequence of statements that end in a conclusion

By valid we mean the conclusion must follow from the truth of the preceding statements or premises

We use rules of inference to construct valid arguments





Valid Arguments in Propositional Logic

When we replace statements/propositions with propositional variables we have an argument form.

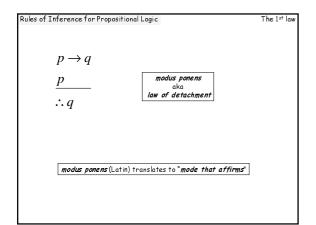
Defn:

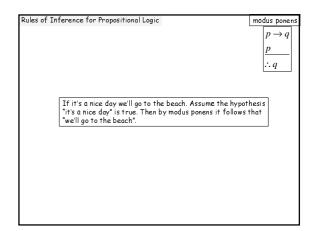
An argument (in propositional logic) is a sequence of propositions.

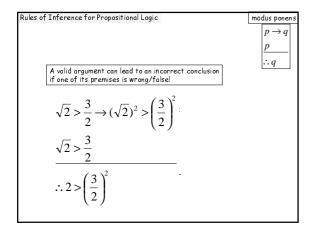
All but the final proposition are called premises.

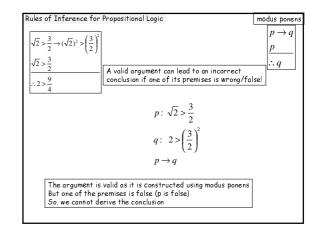
The last proposition is the conclusion
The argument is valid iff the truth of all premises implies the conclusion is true.

An argument form is a sequence of compound propositions









The rules of i	nference			Page 66
	Rule of inference	Tautology	Name	
	$p \rightarrow q$			
	<u>P</u>	$[p \land (p \rightarrow q)] \rightarrow q$	Modus ponens	
	∴ q			
	$\neg q$			
	$\underline{p \rightarrow q}$	$[\neg q \land (p \rightarrow q)] \rightarrow \neg p$	Modus tollen	
	∴ ¬p			
	$p \rightarrow q$			
	$q \rightarrow r$	$[(p \to q) \land (q \to r)] \to (p \to r)$	Hypothetical syllogism	
	$\therefore p \rightarrow r$			
	$p \lor q$			
	<u>¬p</u>	$((p \lor q) \land \neg p) \rightarrow q$	Disjunctive syllogism	
	∴.q			
	<u>p</u>	$p \rightarrow (p \lor q)$	Addition	
	∴ p ∨ q			
	<u>p∧q</u>	$(p \land q) \rightarrow p$	Simplifica tion	
	∴ p			
	g g	$((p) \land (q)) \rightarrow (p \land q)$	Conjunction	
		$((p) \land (q)) \rightarrow (p \land q)$	Conjunction	
	$p \lor q$			
	$\neg p \lor r$	$[(p \lor q) \land (\neg p \lor r)] \rightarrow (p \lor r)$	Resolution	
	_	$\begin{bmatrix} (p \vee q) \wedge (-p \vee r) \end{bmatrix} \rightarrow (p \vee r)$	Resolution	
	∴ q∨r			

Another view on what we are doing

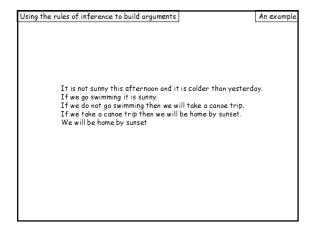
You might think of this as some sort of game.

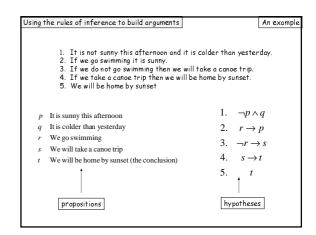
You are given some statement, and you want to see if it is a valid argument and true

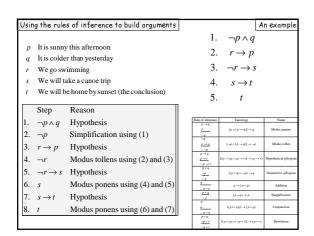
You translate the statement into argument form using propositional variables, and make sure you have the premises right, and clear what is the conclusion

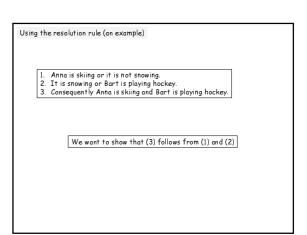
You then want to get from premises/hypotheses (A) to the conclusion (B) using the rules of inference.

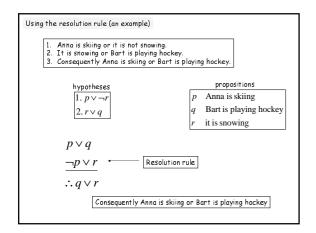
So, get from A to B using as "moves" the rules of inference

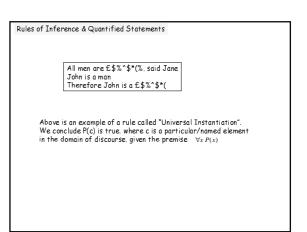




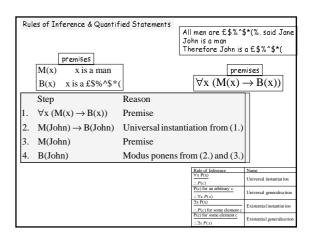








	erence Name
$\frac{\forall x \ P(x)}{\therefore P(c)}$	Universal instantiation
$\frac{P(c) \text{ for an } a}{\therefore \forall x \ P(x)}$	Universal generalisation
$\exists x \ P(x)$ $\therefore P(c) \text{ for so}$	Existential instantiation
$\frac{P(c) \text{ for som}}{\therefore \exists x \ P(x)}$	e element c Existential generalisation



Rules of Inference & Quantified Statements				
Maybe another example?				