Title: Dual Viewpoint Heuristics for Binary Constraint Satisfaction Problems

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Theory and Practice of Constraint Satisfaction

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Contributions

I- New heuristics for variable & value selection

II- Double-viewpoint strategy
(common in scheduling: [Sadeh '91]
job vs. resource-centered perspective)

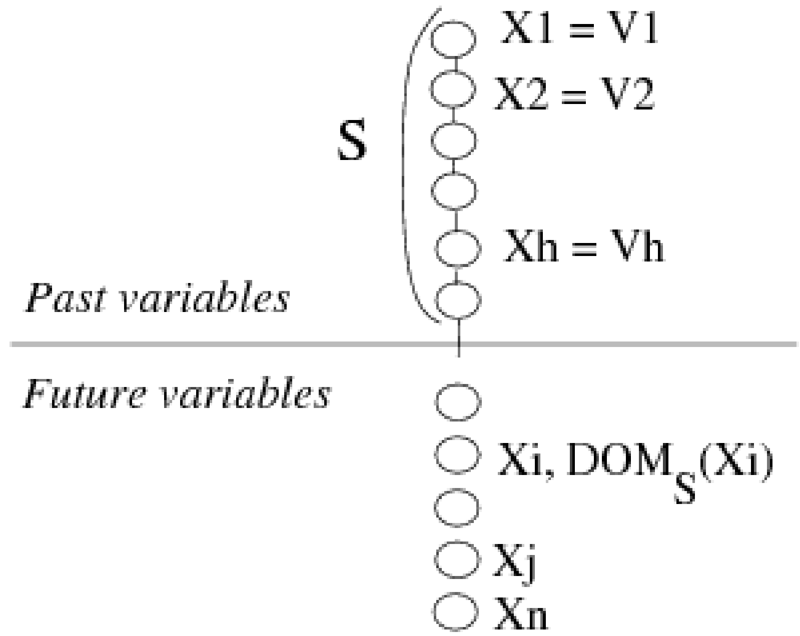
III- Validation on the n-Queen problem

Assumptions

- Binary constraints, finite domains
- Seeking **one** solution (relevant for value ordering)
- Using backtracking (BT) and forward-checking (FC)

I- Goal of Var/Val orderings in BT

- **avoid constraint violation**
  → select values that do not cause constraint violation
  → most promising value first
- **discover constraint violation quickly**
  → select variables that do not delay constraint violation
  → most constrained variable first (fail first principle)



$$\text{After forward-checking given } S \begin{cases} \mathrm{DOM}_S(X_i) \\ \vdots \\ \mathrm{DOM}_S(X_j) \\ \mathrm{DOM}_S(X_n) \end{cases}$$

For $X_i = V_i$, we can separate $\mathrm{DOM}_S(X_j)$ into two sets in $\mathcal{O}(a)$:

- values consistent with $X_i = V_i$
  $\rightarrow$ of size $\mathrm{LEFT}_S(X_j \mid X_i = V_i)$

- values inconsistent with $X_i = V_i$
  $\rightarrow$ of size $\mathrm{LOST}_S(X_j \mid X_i = V_i)$

Value selection for X_i

Choose the least constraining value V_i

1. Minimize X_j future variables

$$\text{Cost}_S(X_i = V_i) = \sum_{X_{j \neq i}} \text{LOST}_S(X_j \mid X_i = V_i)$$

2. Minimize X_j future variables

$$\text{Cruciality}_S(X_i = V_i) = \sum_{X_{j \neq i}} \frac{\text{LOST}_S(X_j \mid X_i = V_i)}{|\text{DOM}_S(X_j)|}$$

3. Maximize X_j future variables

$$\text{Promise}_S(X_i = V_i) = \prod_{X_{j \neq i}} \text{LEFT}_S(X_j \mid X_i = V_i)$$

$\rightarrow$ number of assignments that $X_i = V_i$ and can be done such that no constraint on X_i is violated

(3) is more discriminating ($\text{LEFT}_S(X_j \mid X_i = V_i) = 0$)

Value selection: example

- Minimize X_j future variables

$$\text{Cost}_S(X_i = V_i) = \sum_{X_{j \neq i}} \text{LOST}_S(X_j \mid X_i = V_i)$$

X1	6	6	6	6
X2	6	8	8	6
X3	6	8	8	6
X4	6	6	6	6

- Maximize X_j future variables

$$\text{Promise}_S(X_i = V_i) = \prod_{X_{j \neq i}} \text{LEFT}_S(X_j \mid X_i = V_i)$$

X1	8	6	6	8
X2	8	2	2	8
X3	8	2	2	8
X4	8	6	6	8

Variable selection

Choose the most constrained variable (FFP) X_i

1. Least domain (LD)

2. Maximize $\qquad V_i \in \mathrm{DOM}_S(X_i)$

   $\mathrm{Criticality}_S(X_i) =$

   $$\prod_{V_i} \frac{1}{(1+|\mathrm{DOM}_{S(X_i)}|+\mathrm{Cruciality}_{S(X_i=Vi)})}$$

3. Minimize $\qquad V_i \in \mathrm{DOM}_S(X_i)$

   $$\mathrm{Promise}_S(X_i) = \sum_{V_i} \mathrm{Promise}_S(X_i = V_i)$$

   $\rightarrow$ number of assignments that can be done such that no constraint on X_i is violated

Variable selection: example

Minimize: $V_i \in \text{DOM}_S(X_i)$

$$\text{Promise}_S(X_i) = \sum_{V_i} \text{Promise}_S(X_i = V_i)$$

X1	8	6	6	8	28
X2	8	2	2	8	20
X3	8	2	2	8	20
X4	8	6	6	8	28

X_1, X_4 promise 28 solutions

X_2, X_3 promise 20 solutions, more constraining

Start with X_2 or X_3 (more constraining) and, choose columns 1 or 4 (more promising)

Summary

Most promising value:

(1) Minimum cost

(2) Minimum cruciality

(3) Maximum promise

Most constrained variable:

- Least domain (LD)

(4) Maximum criticality

(5) Minimum (false) promise

$\longrightarrow$ Dynamic variable/value orderings

Algorithms

Identifier	Choice of Var	Choice of val
LD+1	least domain	Minimum cost
LD+2	–	Min. cruciality
LD+3	–	Max promise
FE+2/4	Max. critical	Min. cruciality
FE+3/5	Min. promise	Max. promise

LD: time $\mathcal{O}(na^2)$, space $\mathcal{O}(na)$

FE: time $\mathcal{O}(n^2a^2)$, space $\mathcal{O}(na)$

Implementation hack: domino effect, saves computations.

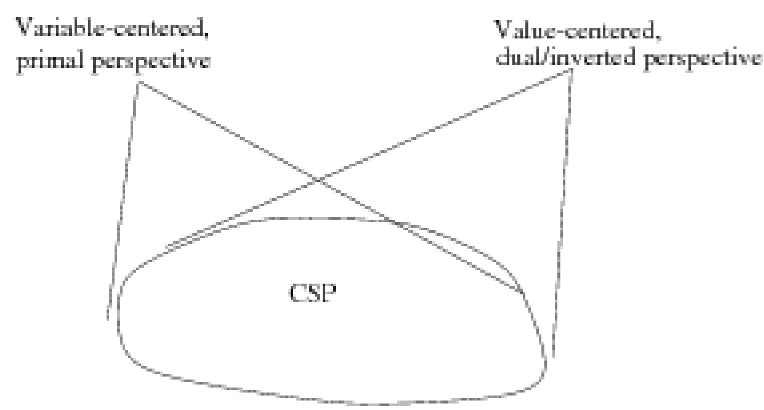
II- Permutation problems

$$\rightarrow \begin{cases} n = a \\ \text{constraint graph is complete} \\ \text{constraints are MUTEX, All-diffs} \end{cases}$$

→ matching, marriage problem

find one value for every variable and
exactly one variable for every value

Viewpoints: variable vs. values



→ Which viewpoint to take?

→ How to combine computations in viewpoints?

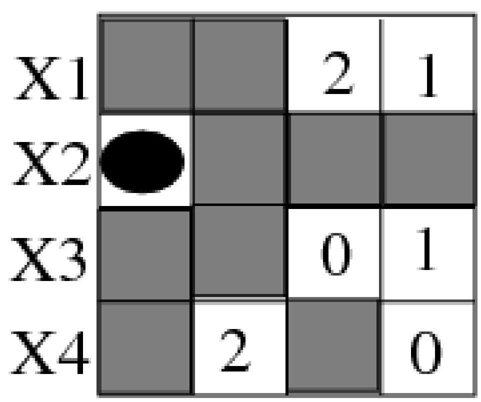
Permutation problems (cont'd)

At any point in BT,

$$\#\text{ future variables} = \#\text{ future values}$$

$\rightarrow$ choose the most constrained variable in the var-viewpoint, except when the most constrained value in the val-viewpoint is more constrained

PP: Least domain



PP: Full evaluation

Introduce: $\begin{cases} \text{LEFT}^{inv}(V_i|X_i = V_i) \\ \text{Promise}^{inv}(V_i = X_i) \end{cases}$

Combine viewpoints: $\text{CPromise}(X_i = V_i)$ min. $\begin{cases} \text{Promise}(X_i = V_i) \\ \text{Promise}^{inv}(V_i = X_i) \end{cases}$

Evaluate vars/vals using: $\begin{cases} \text{CPromise}(X_i) = \sum_k \text{CPromise}(X_i = V_k) \\ \text{CPromise}^{inv}(V_i) = \sum_k \text{CPromise}(X_k = V_i) \end{cases}$

Partial permutation problems ($n \leq a$)

$\rightarrow$ fake a real PP with bogus variables

$\rightarrow$ extend (3) for PPP

III- Experiments

- 100 N-queen problems: $4 \leq N \leq 103$
- Comparison criteria
  - average number of backtrack
  - number of backtrack-free solutions
  - maximum number of backtracks
  - number of constraint checks

$$\text{Algorithms:} \begin{cases} \text{LD-1, LD-2, LD-3, LD-1+Dual} \\ \text{FE-2-4, FE-3-5, FE-3-5+Dual} \\ \text{FP-2-4, FP-3-5} \end{cases}$$

Include Table from paper

Dual perspective exhibits dramatic improvements