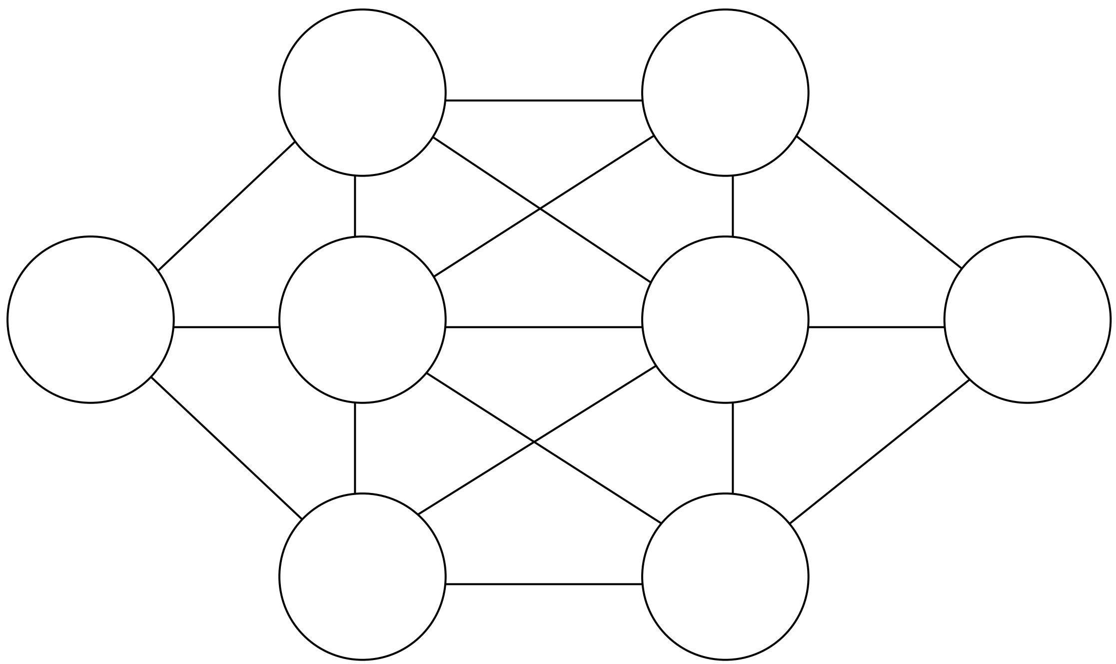


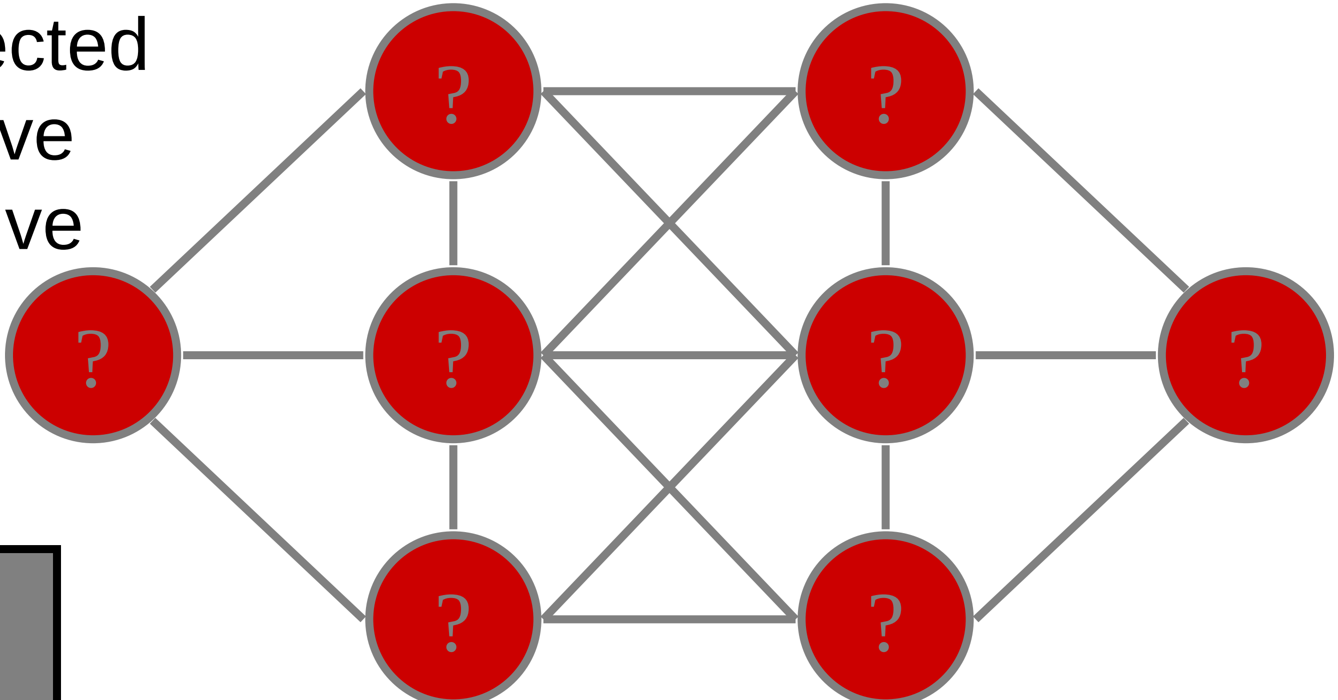
Put a different number in each circle (1 to 8) such that adjacent circles cannot take consecutive numbers

Constraint Programming
An Introduction
by example

with help from Toby Walsh, Chris Beck,
Barbara Smith, Peter van Beek, Edward Tsang, ...

A Puzzle

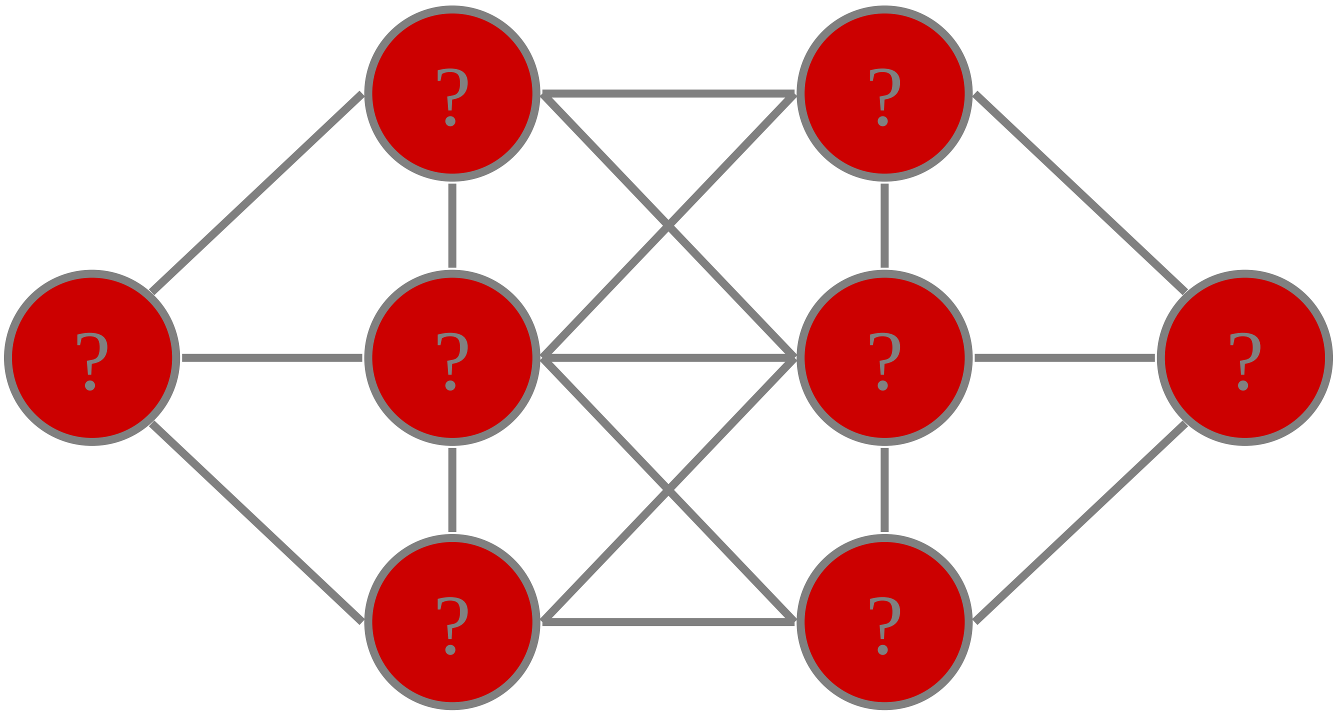
- Place numbers 1 through 8 on nodes
  - Each number appears exactly once
  - No connected nodes have consecutive numbers



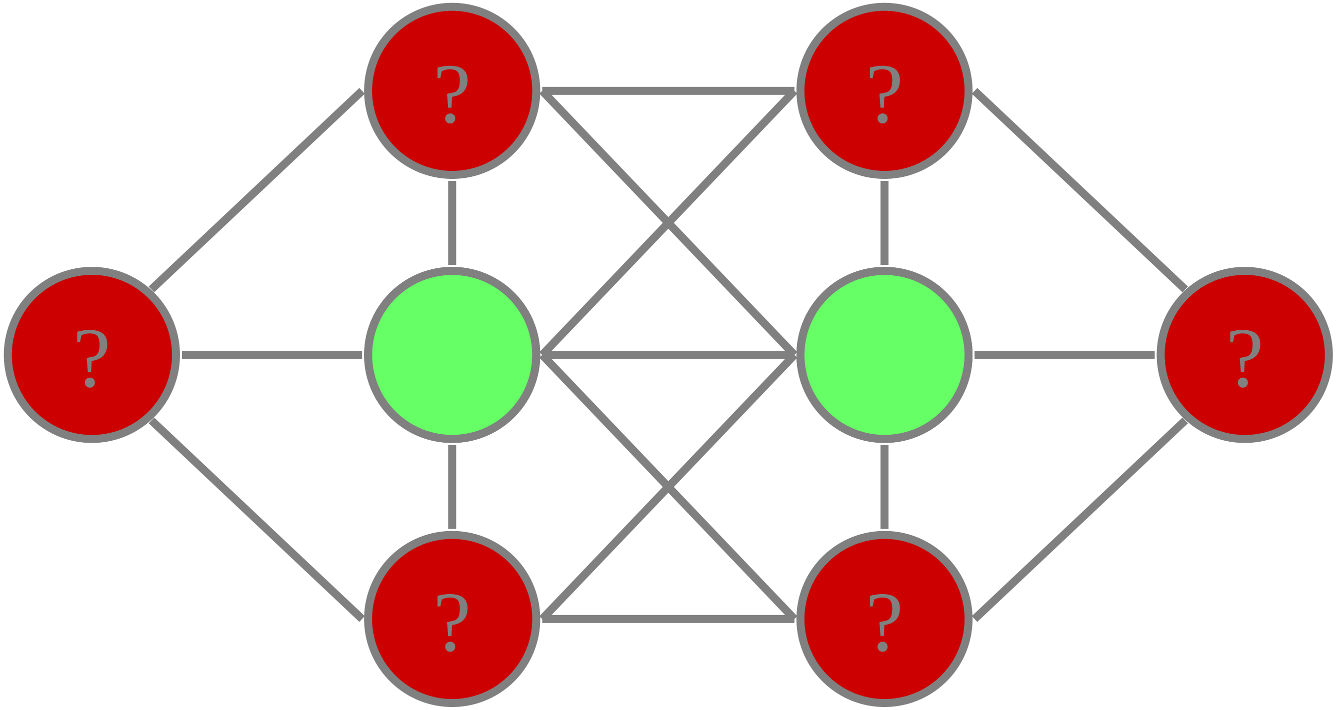
You have
8 minutes!

Heuristic Search

Which nodes are hardest to number?

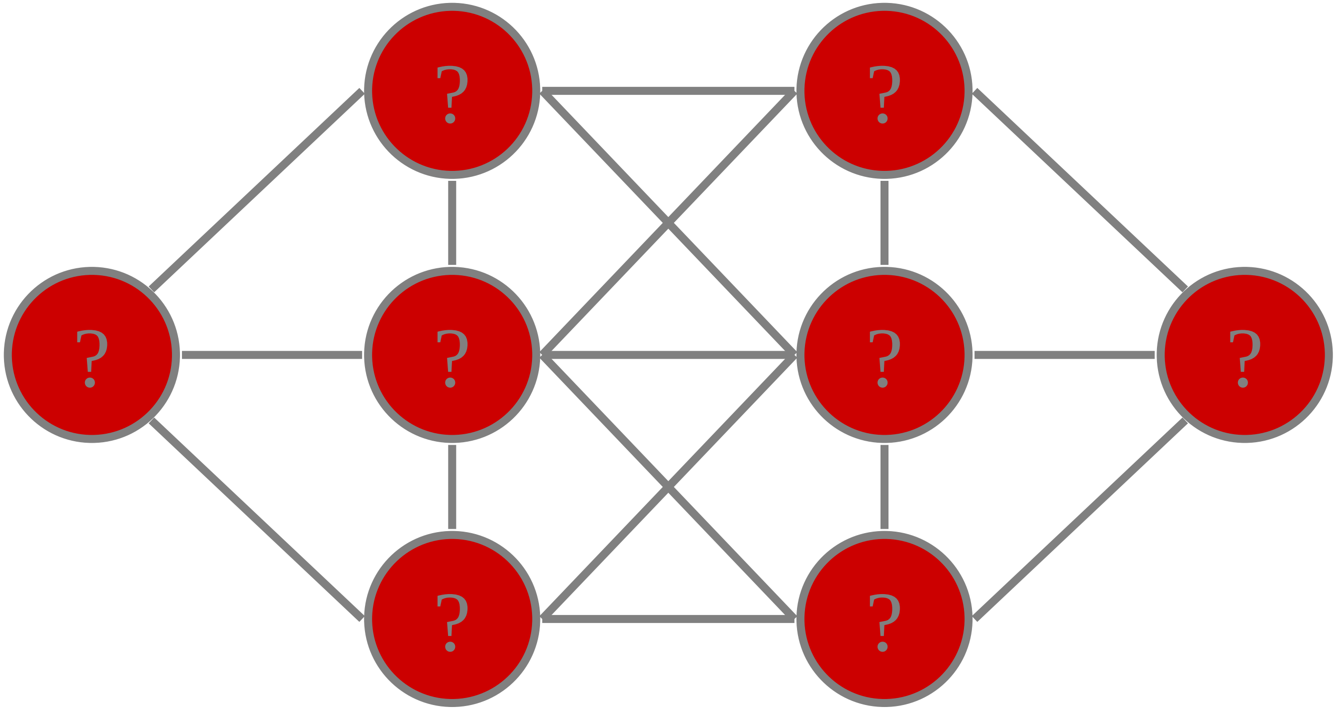


Heuristic Search



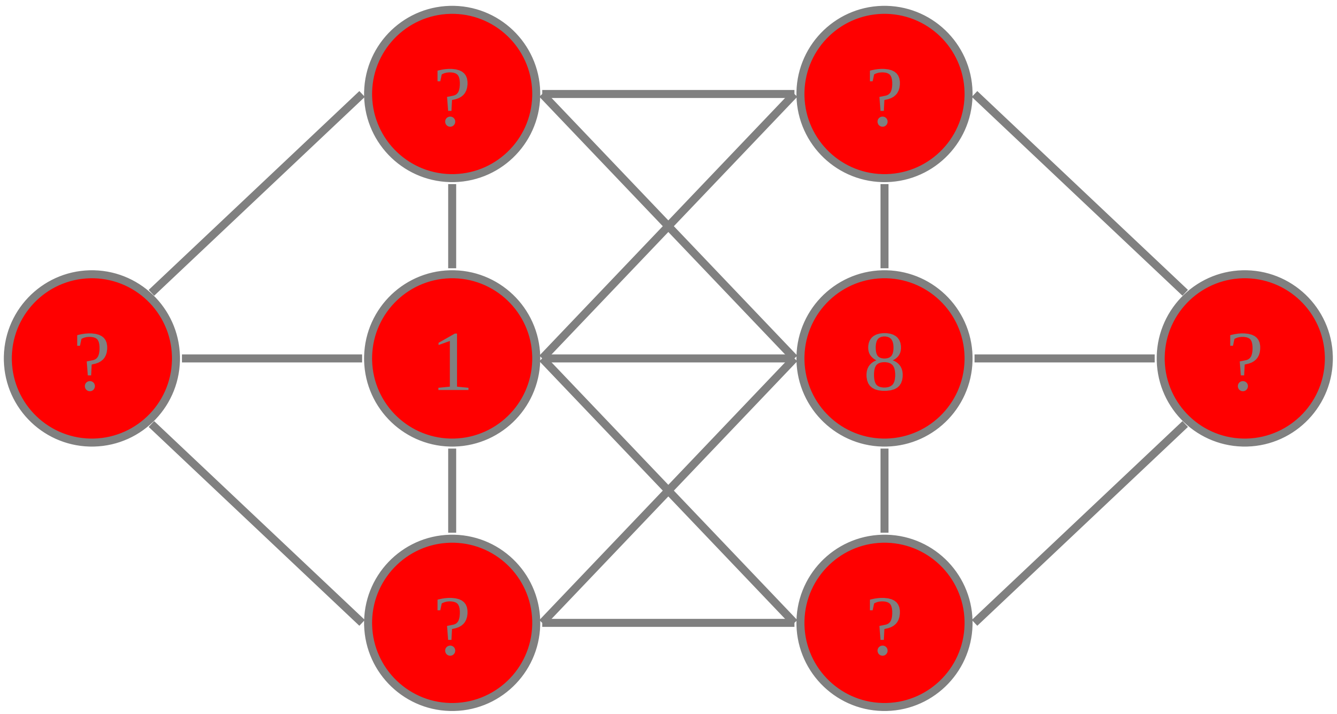
Heuristic Search

Which are the least constraining values to use?



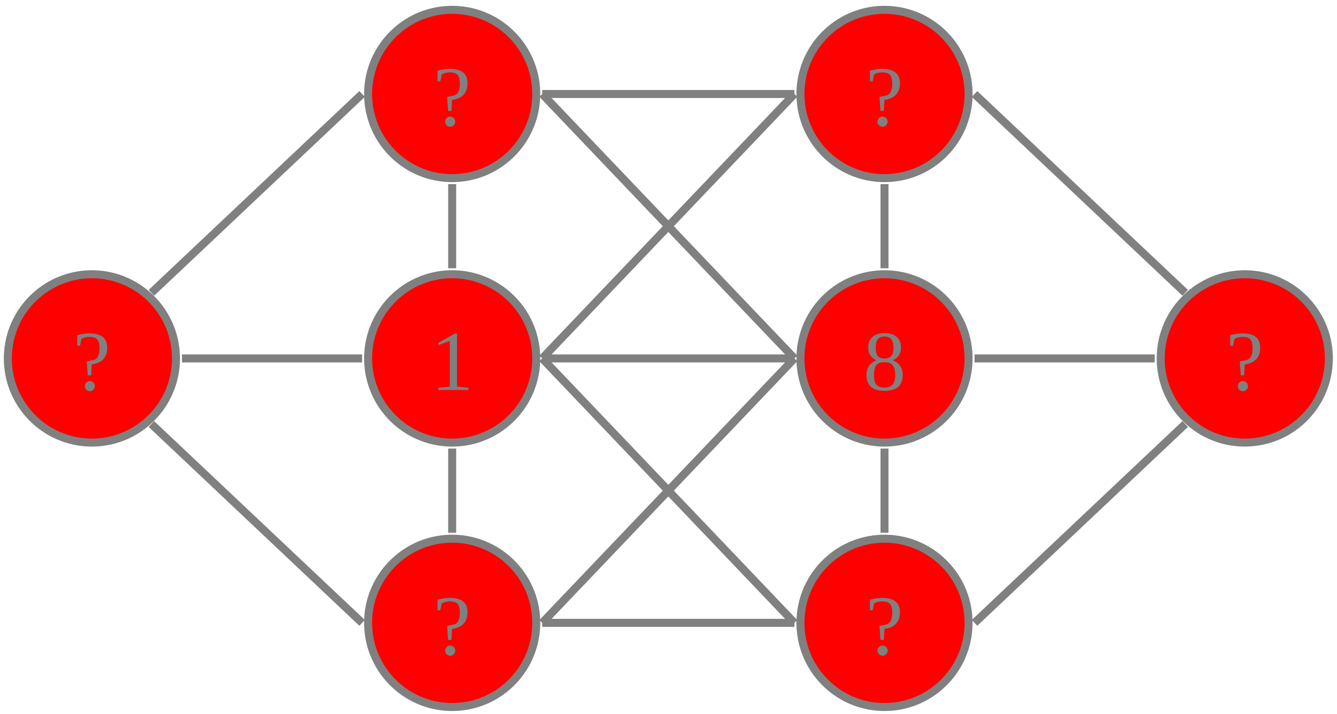
Heuristic Search

Values 1 and 8



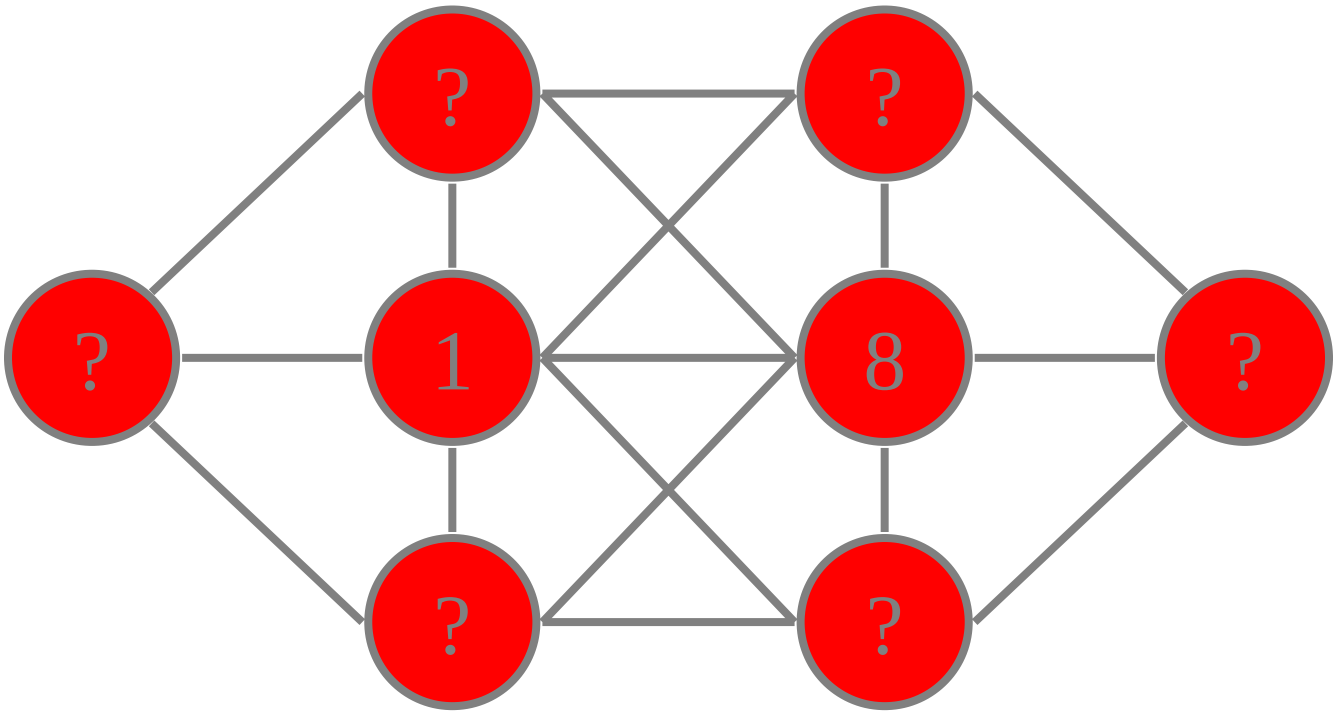
Heuristic Search

Values 1 and 8



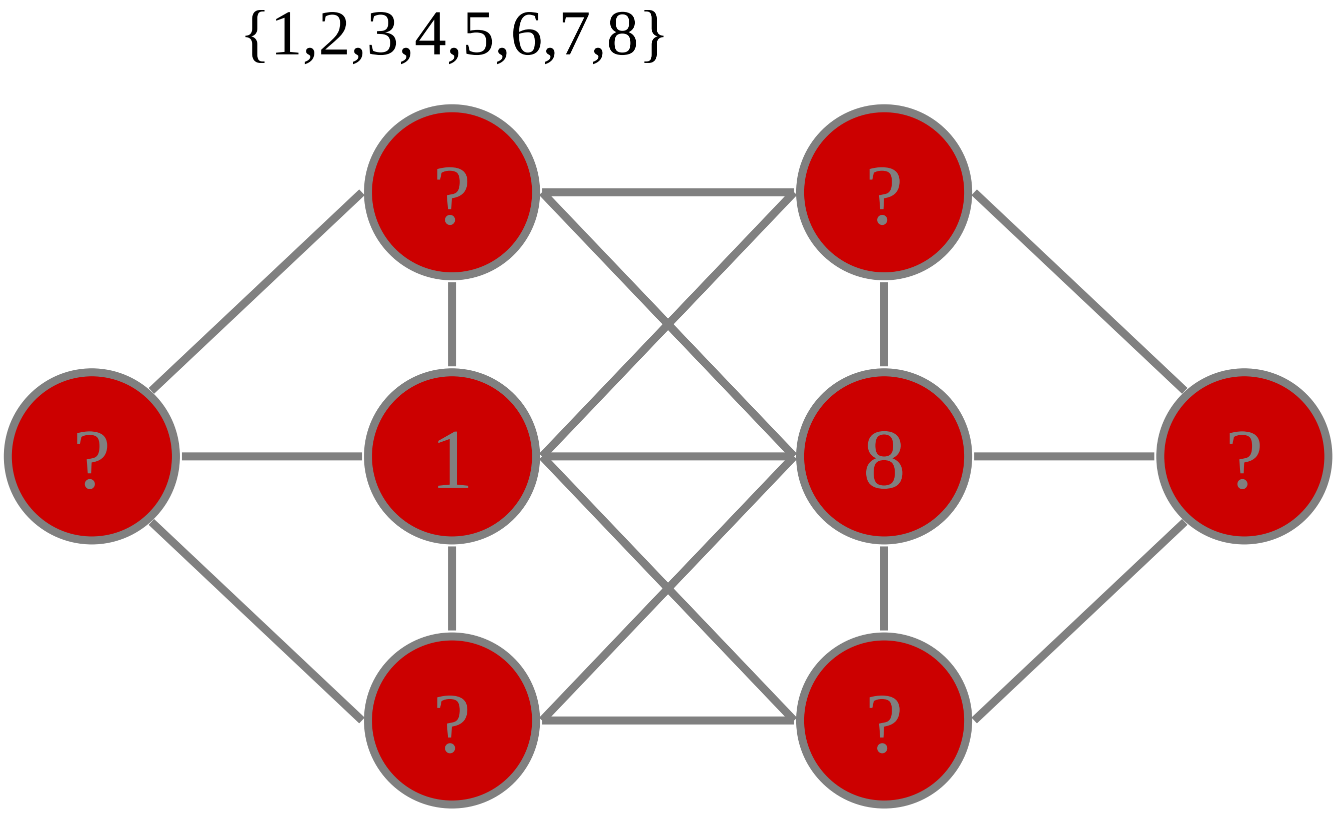
Symmetry means we don't need to consider: 8 1

Inference/propagation

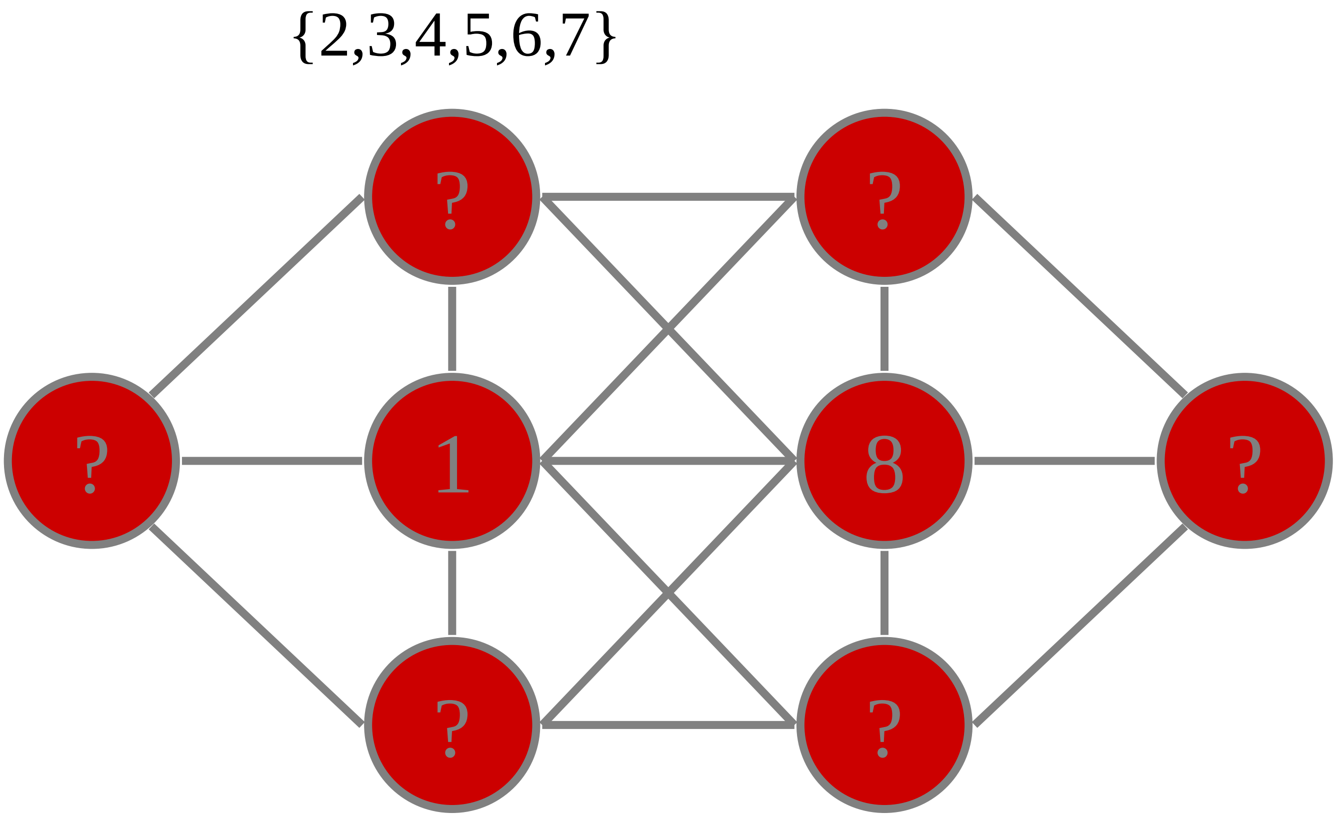


We can now eliminate many values for other nodes

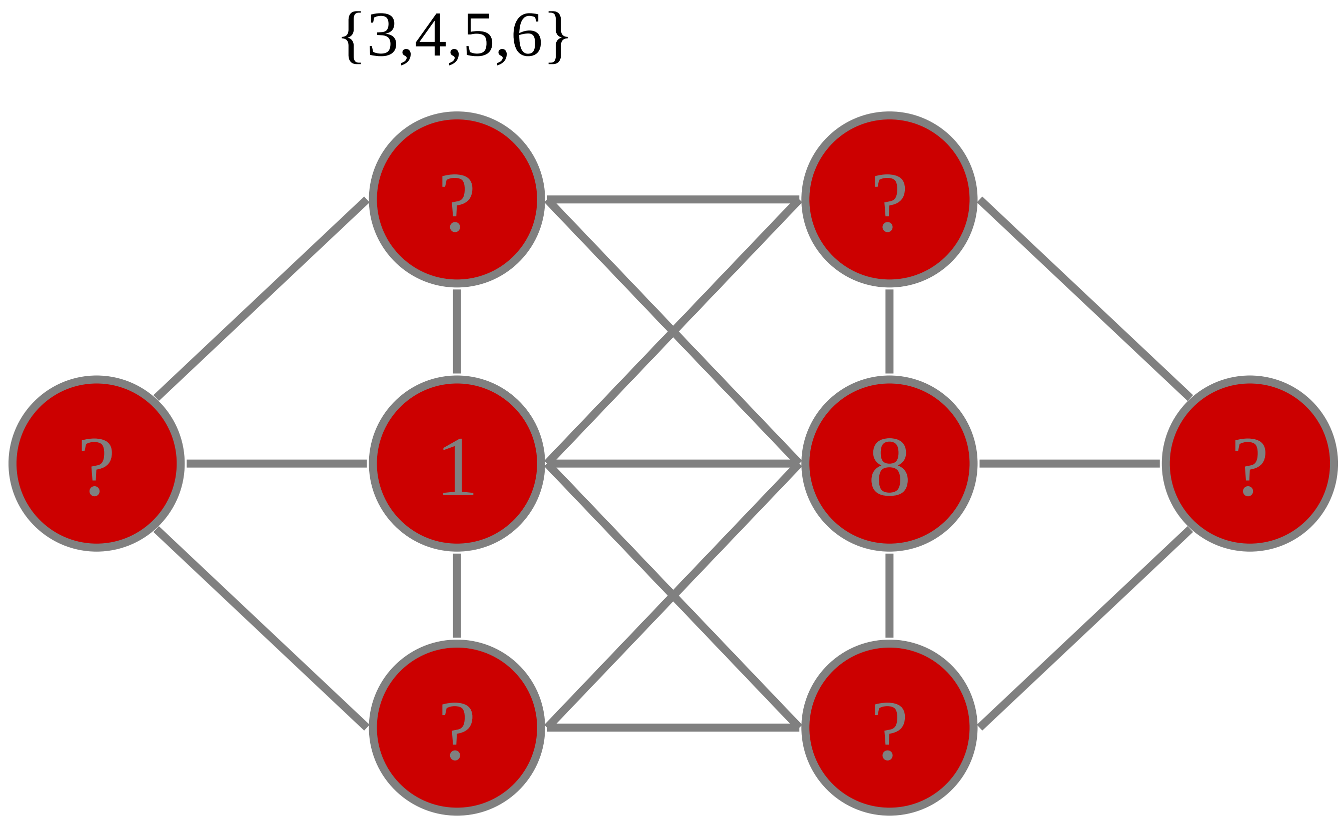
Inference/propagation



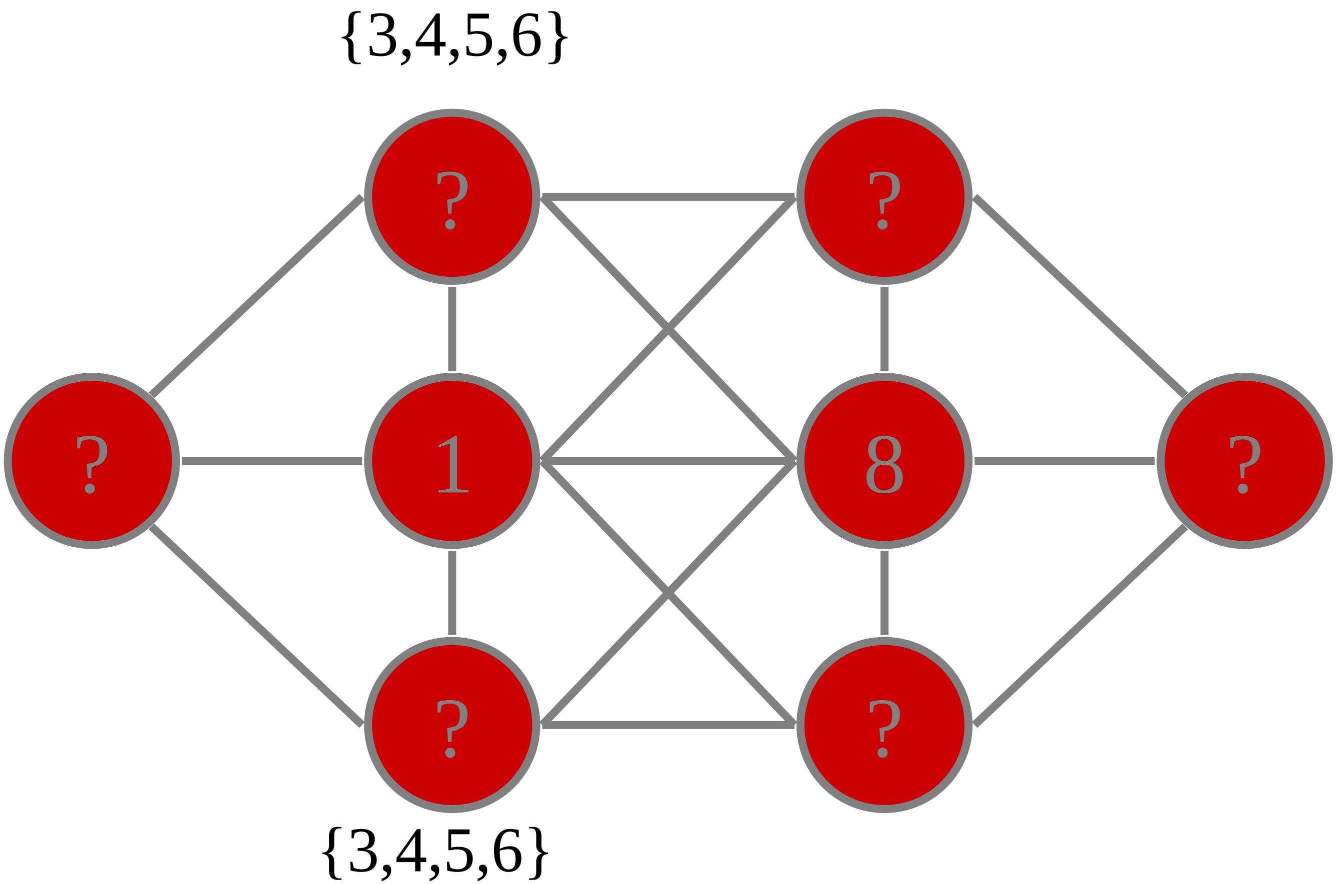
Inference/propagation



Inference/propagation

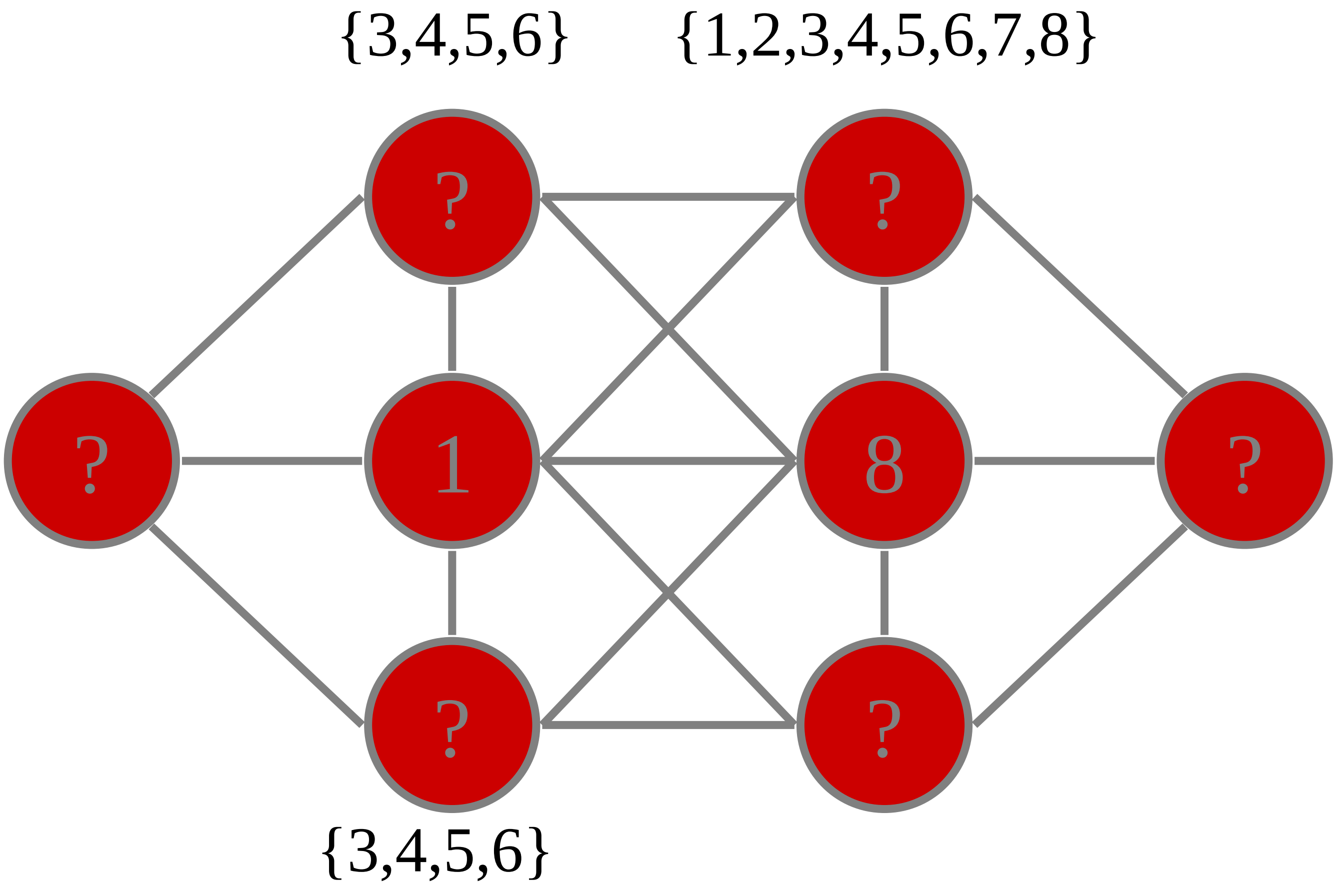


Inference/propagation

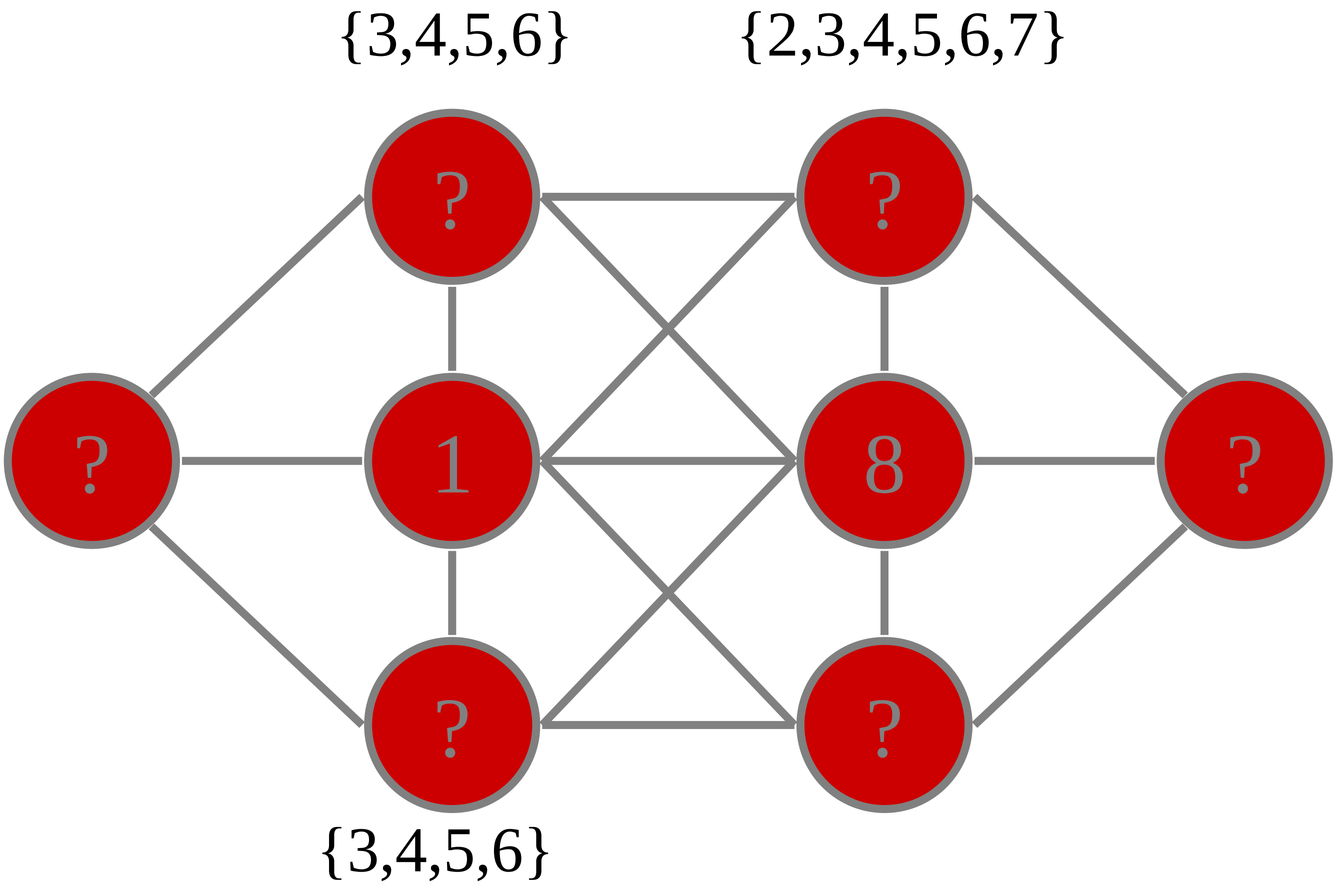


By symmetry

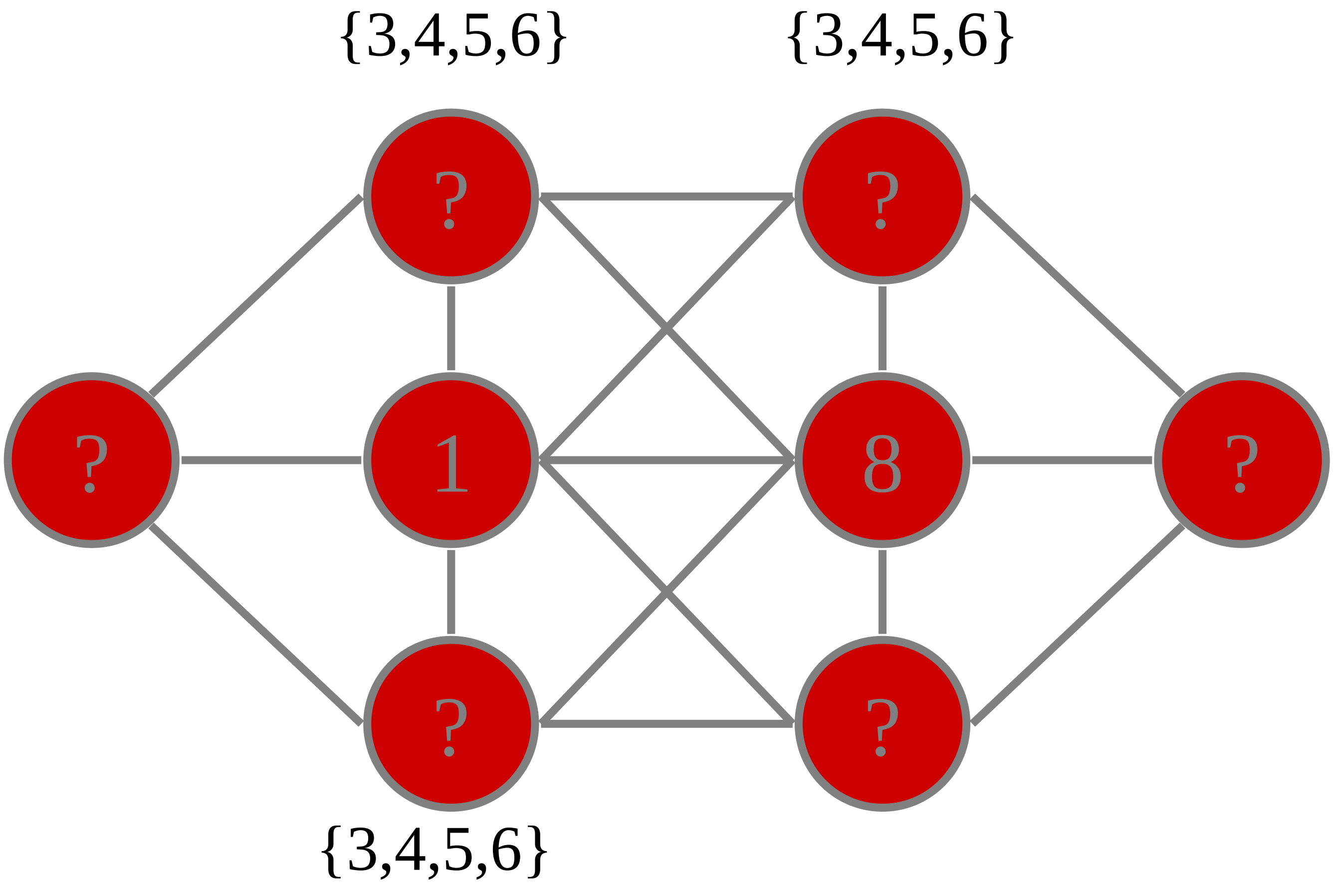
Inference/propagation



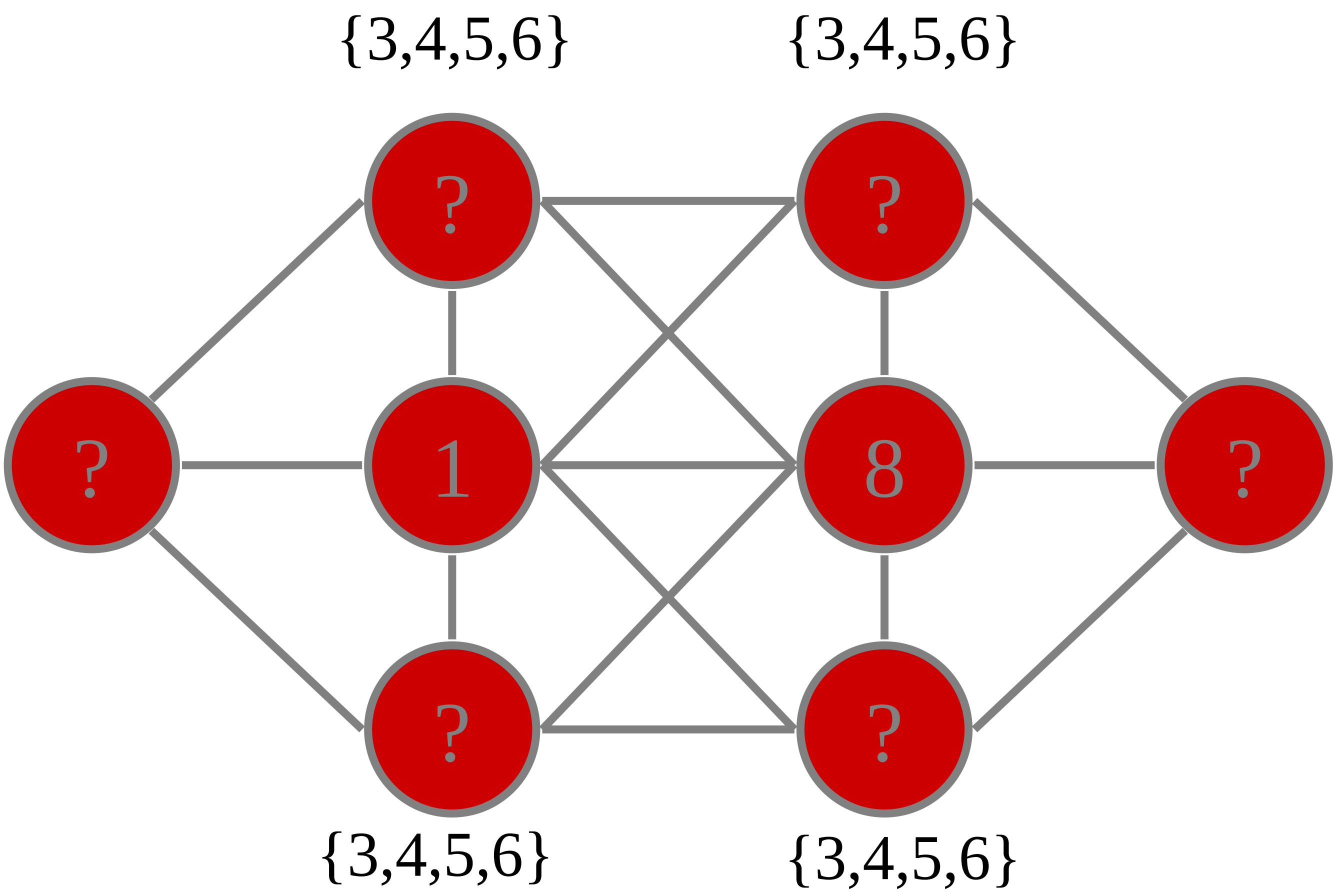
Inference/propagation



Inference/propagation

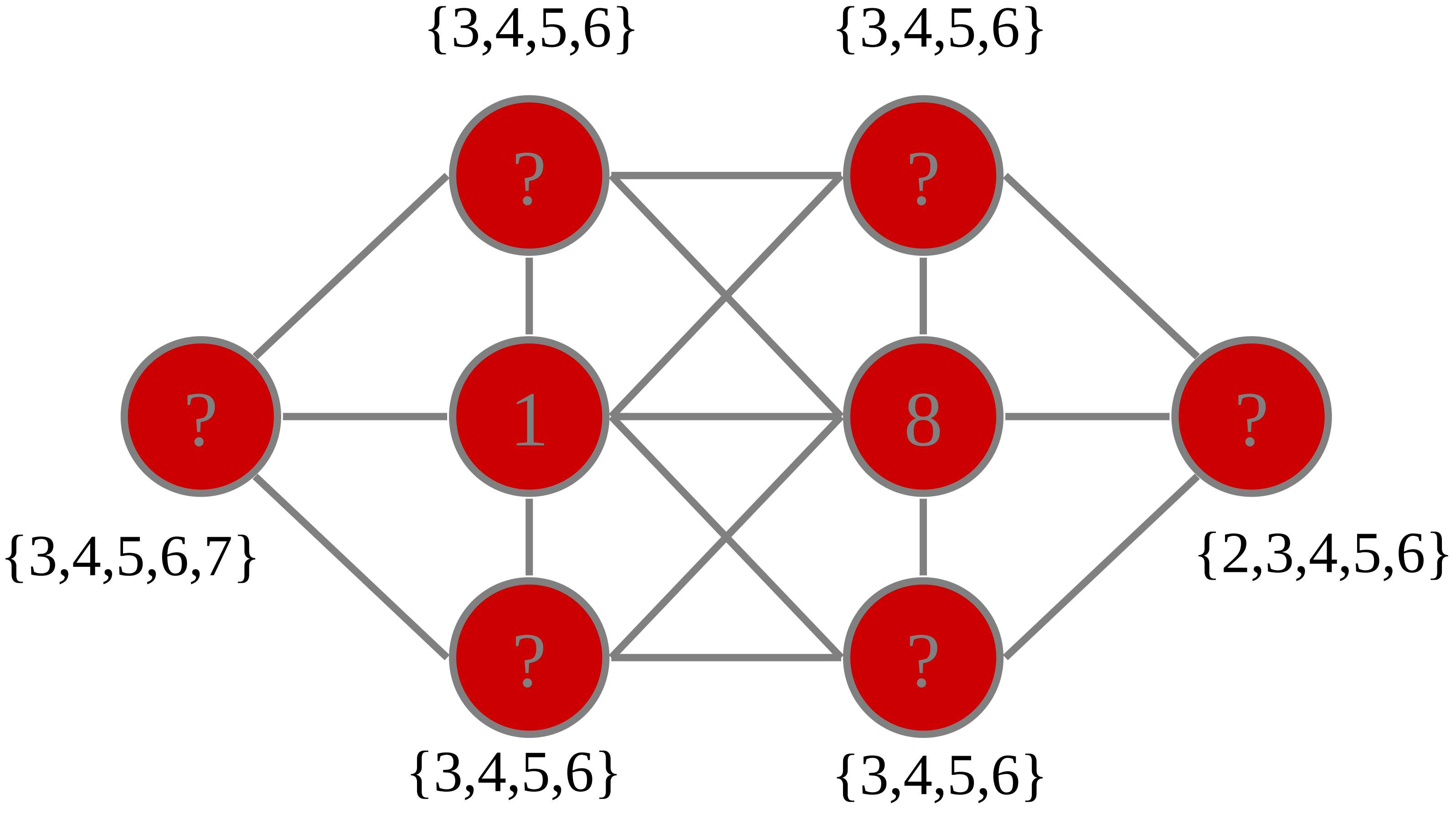


Inference/propagation

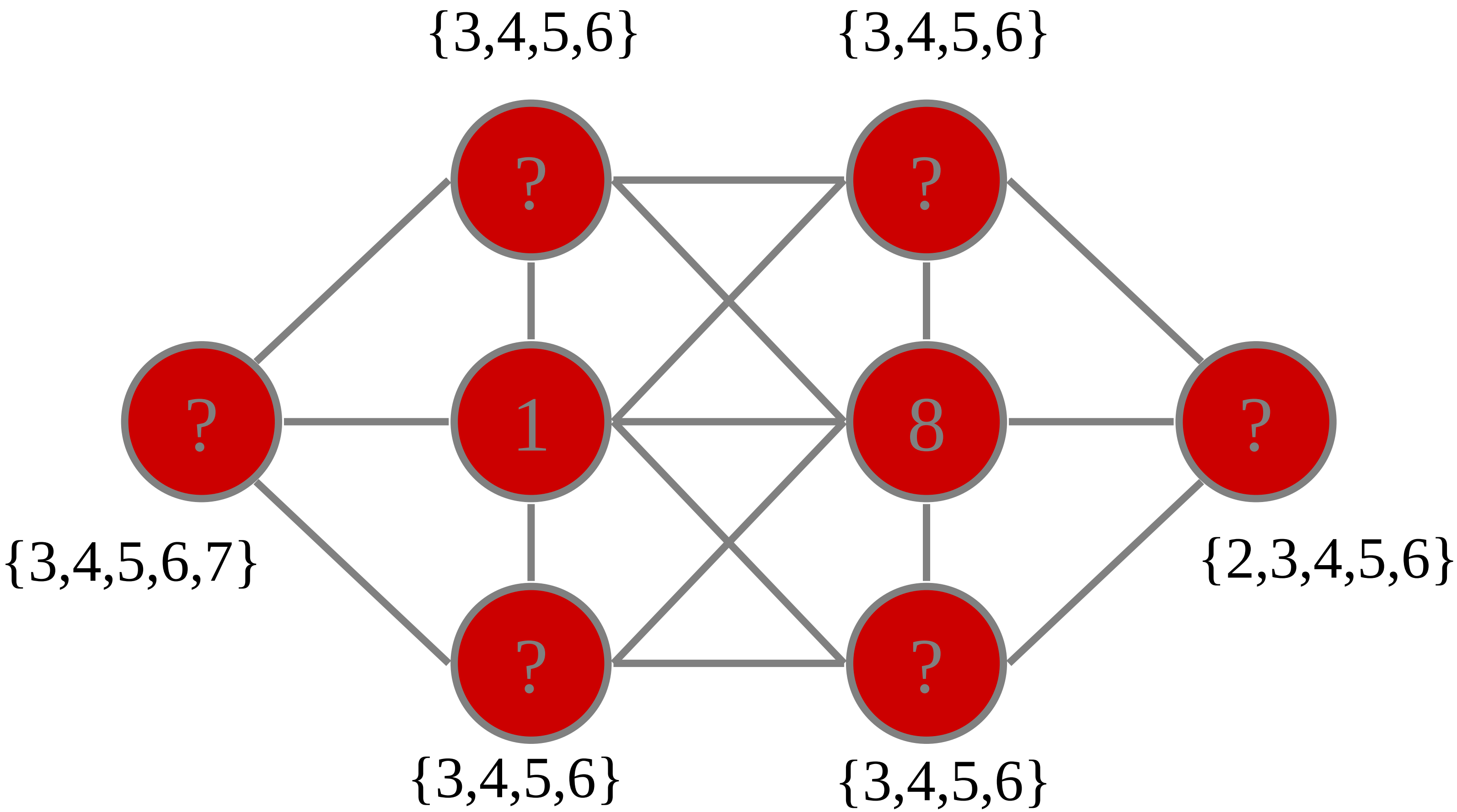


By symmetry

Inference/propagation

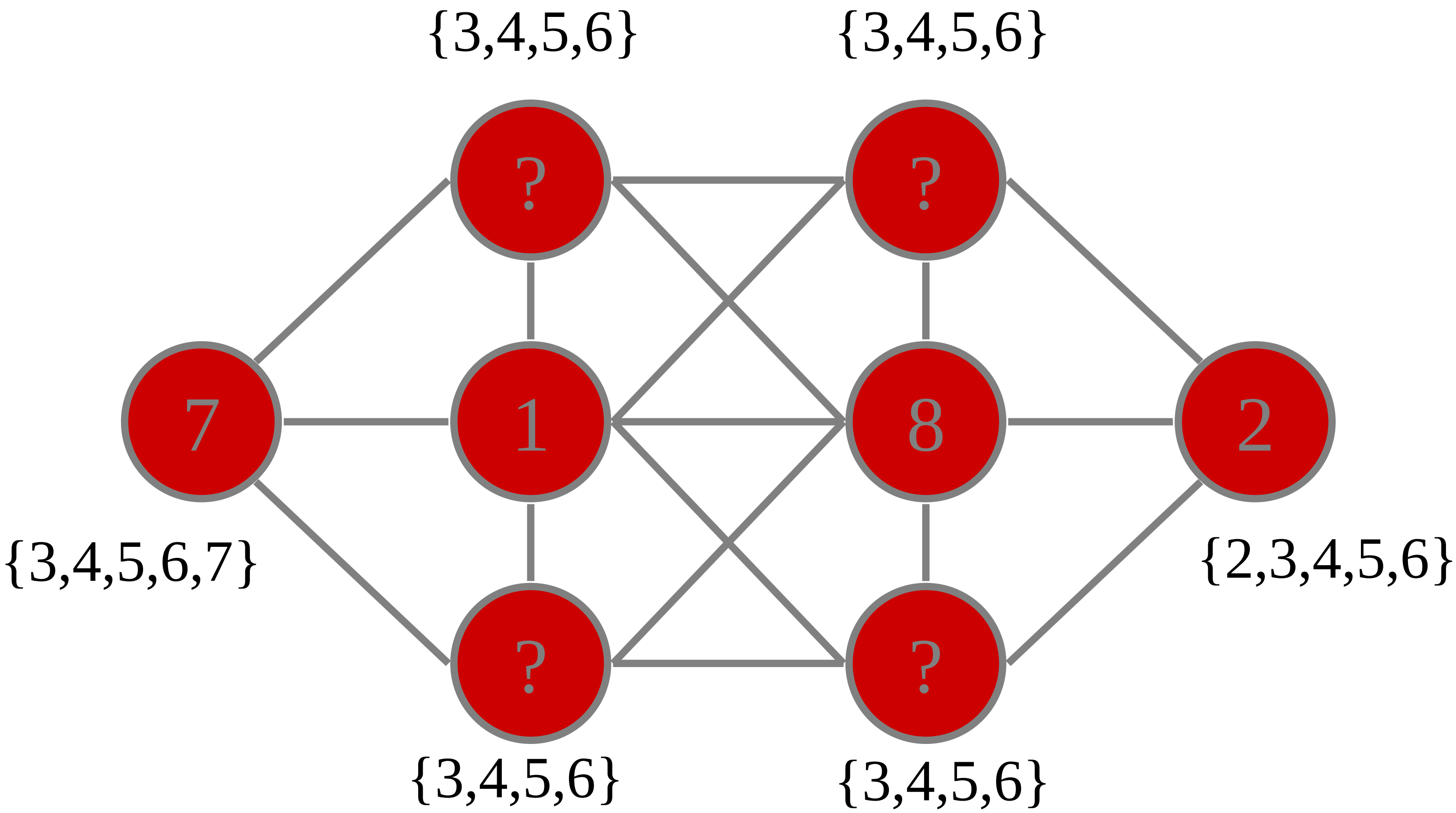


Inference/propagation



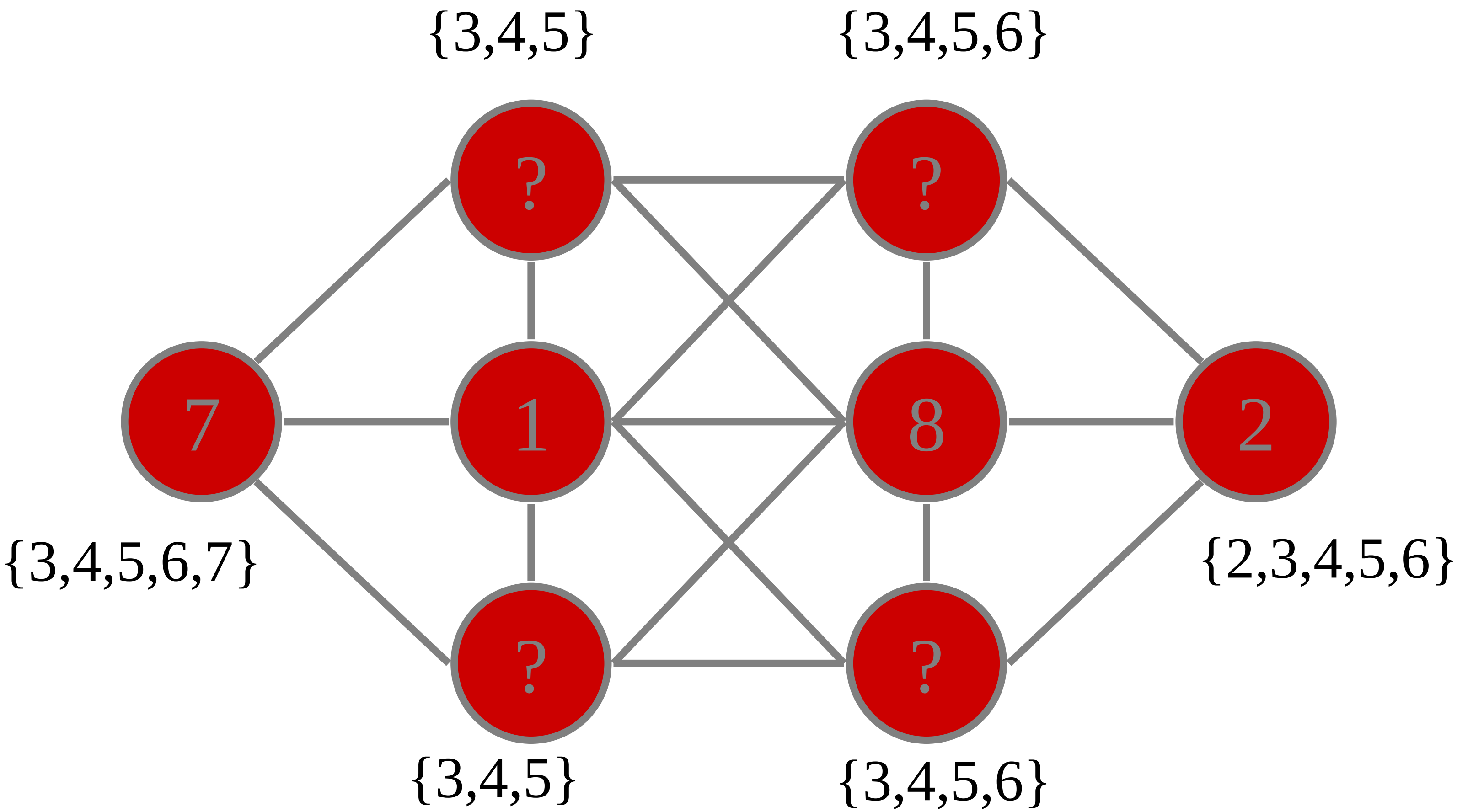
Value 2 and 7 are left in just one variable domain each

Inference/propagation



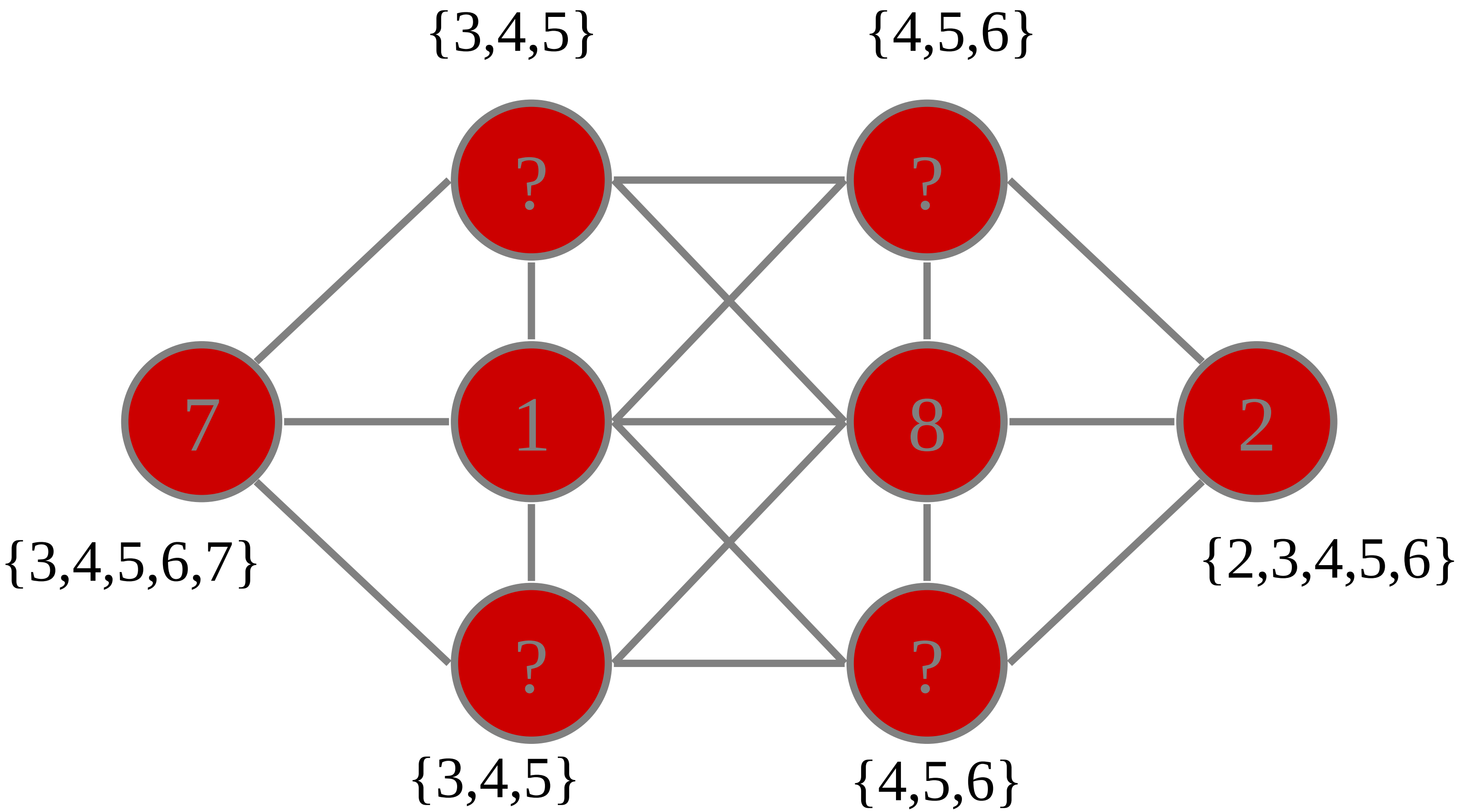
And propagate …

Inference/propagation



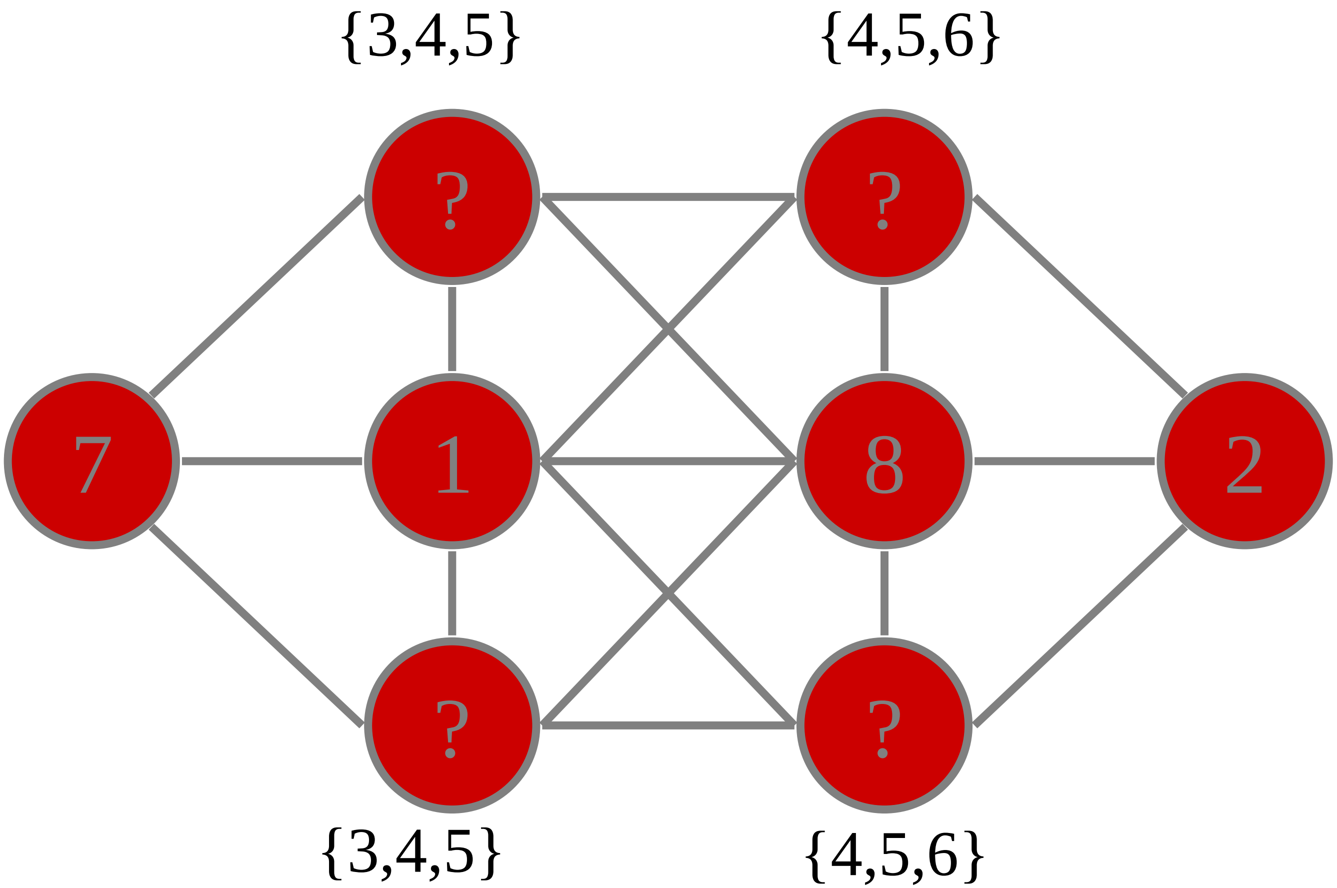
And propagate …

Inference/propagation



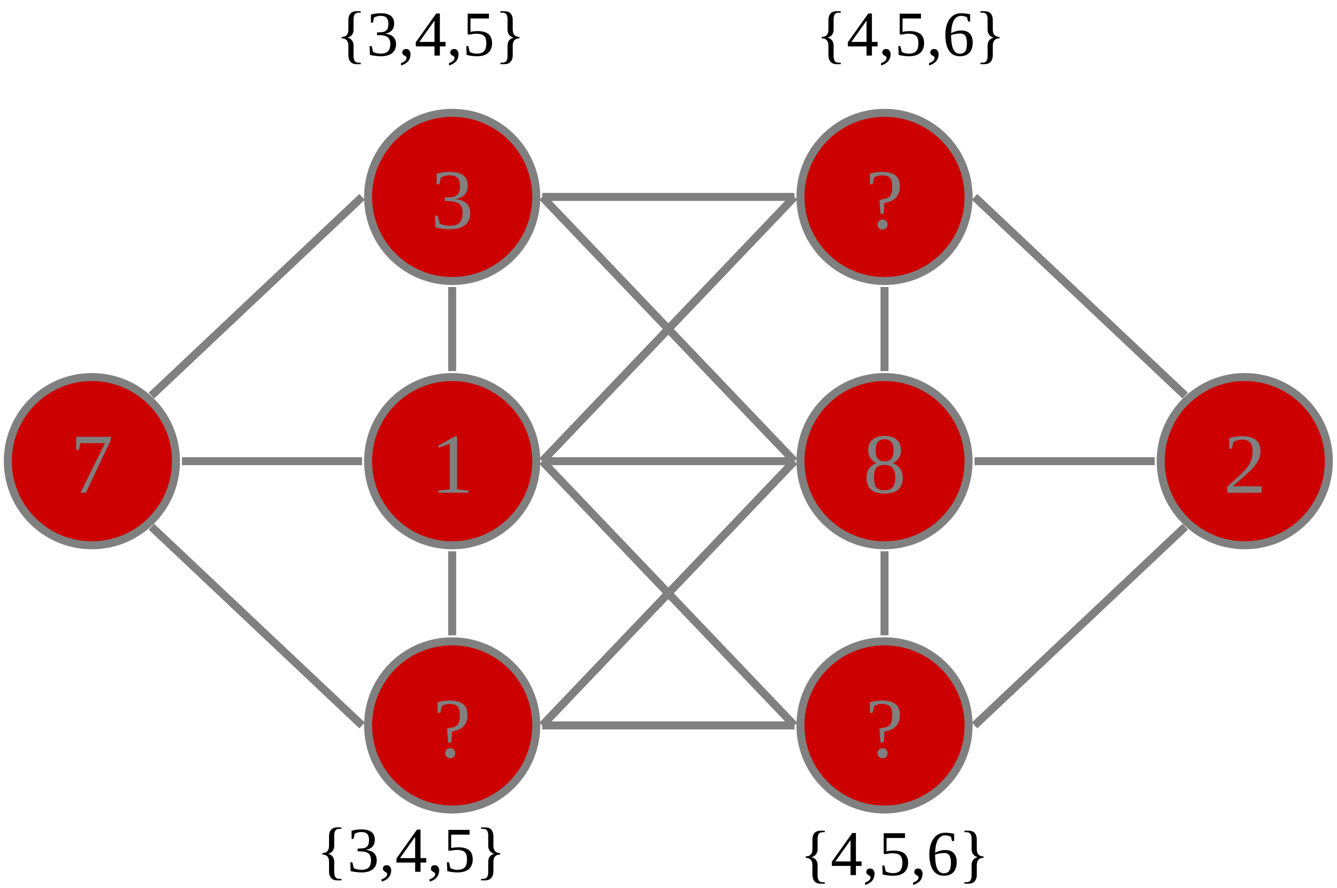
And propagate …

Inference/propagation



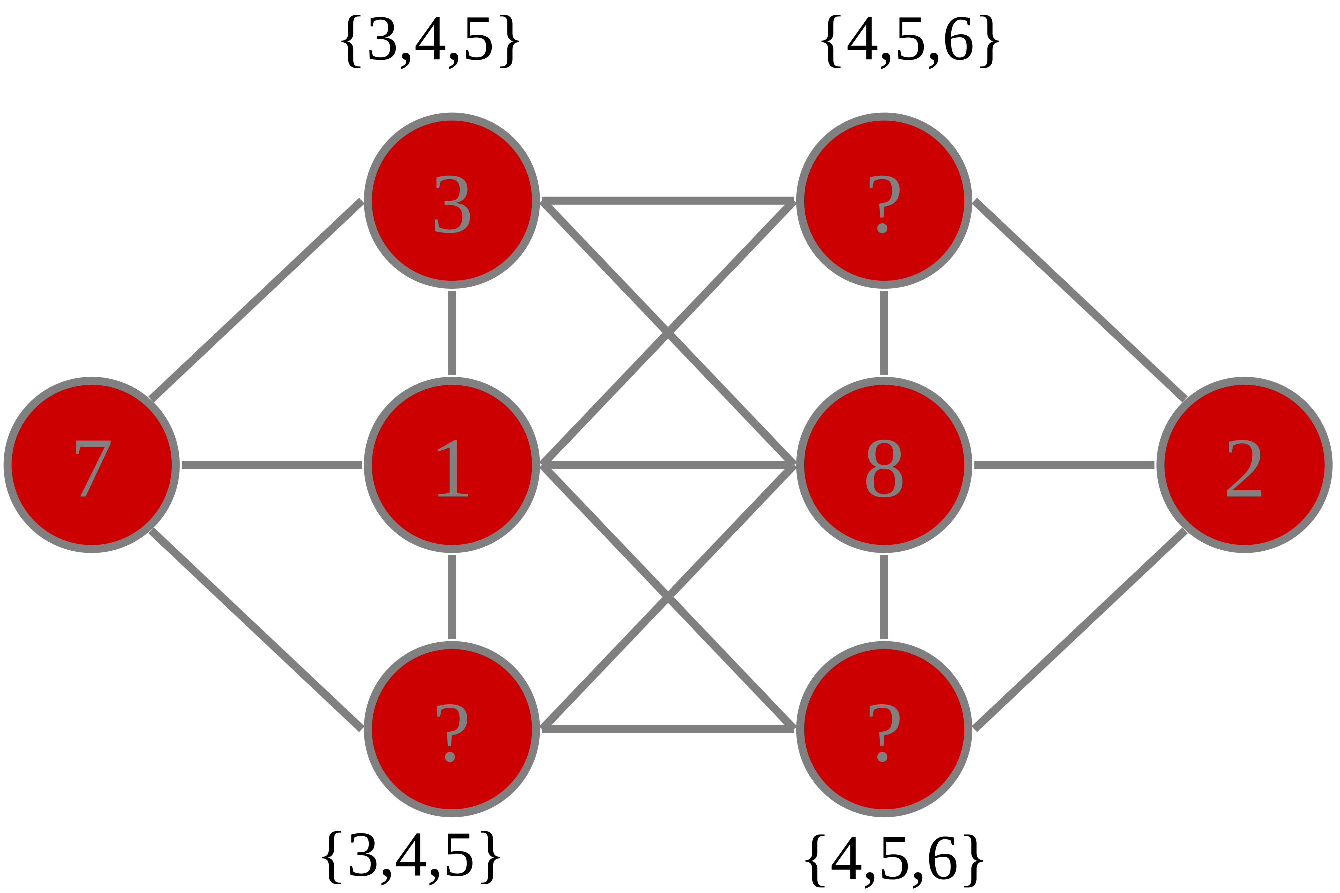
Guess a value, but be prepared to backtrack …

Inference/propagation



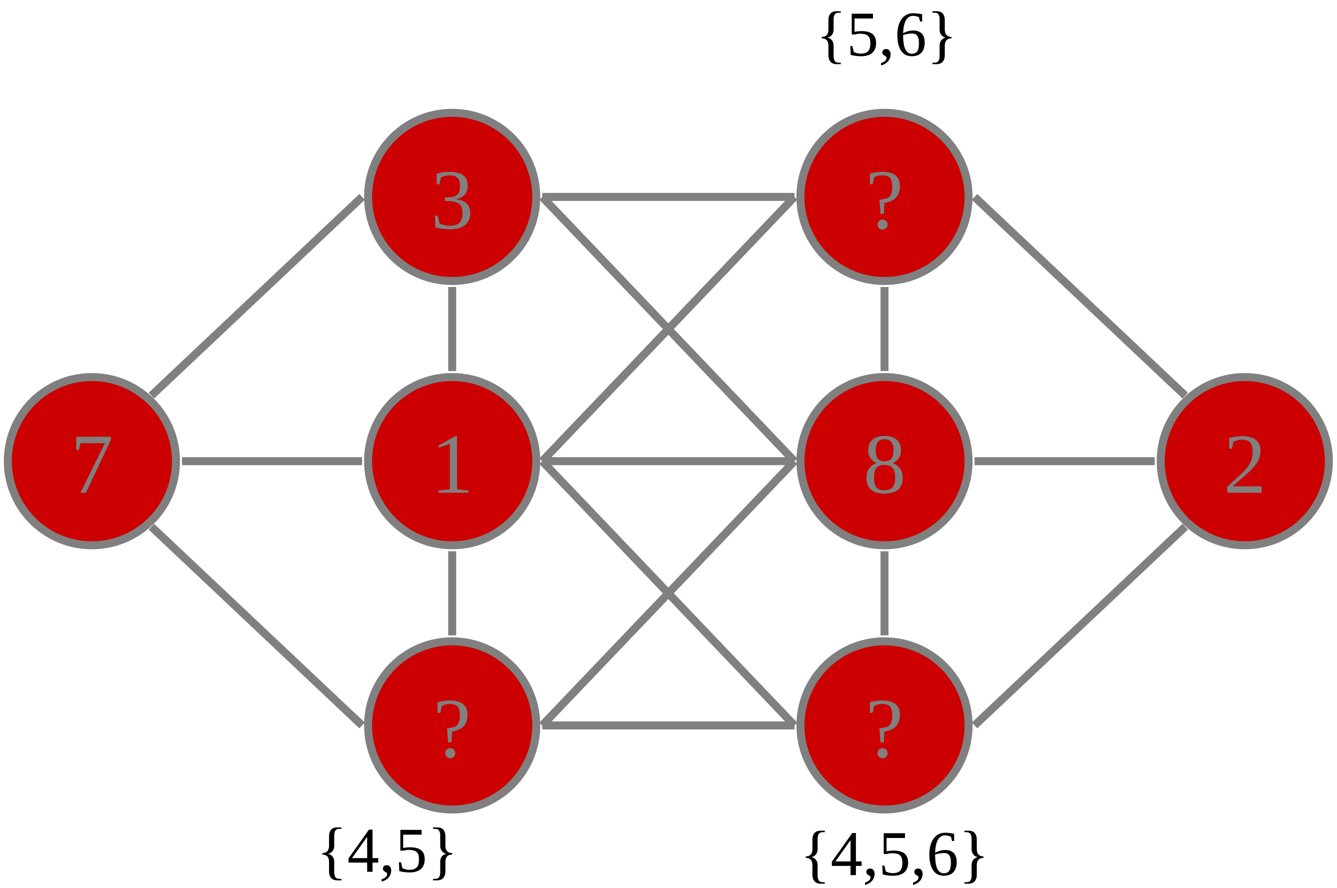
Guess a value, but be prepared to backtrack …

Inference/propagation



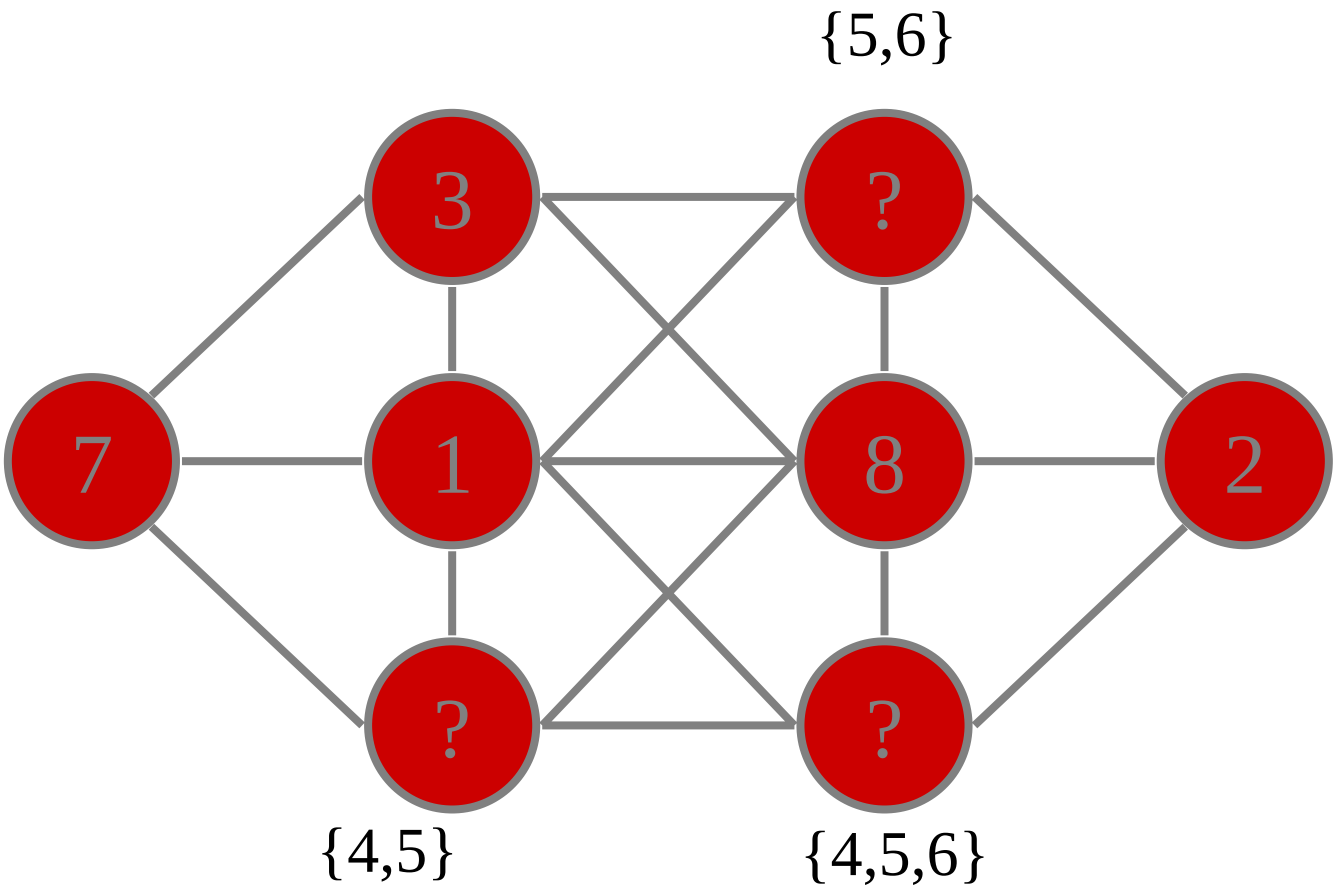
And propagate …

Inference/propagation



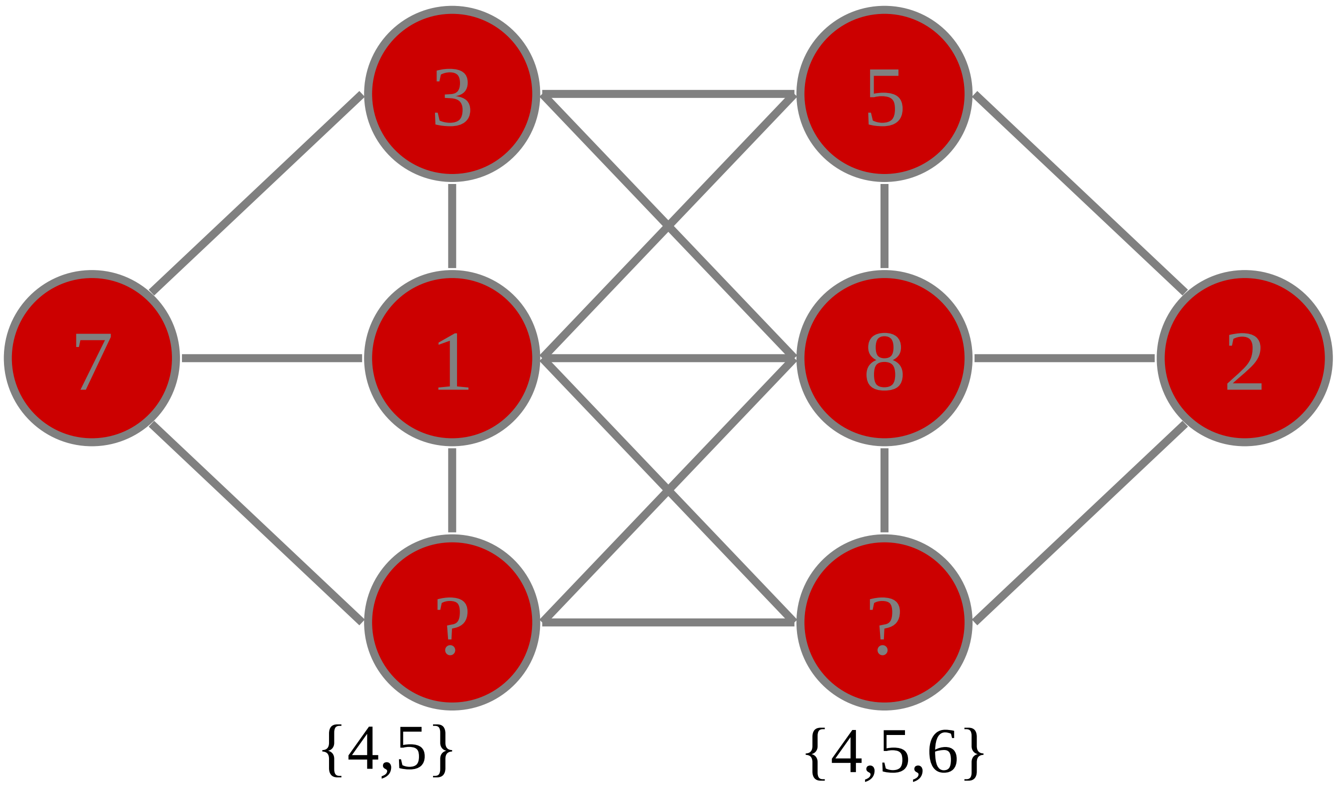
And propagate …

Inference/propagation



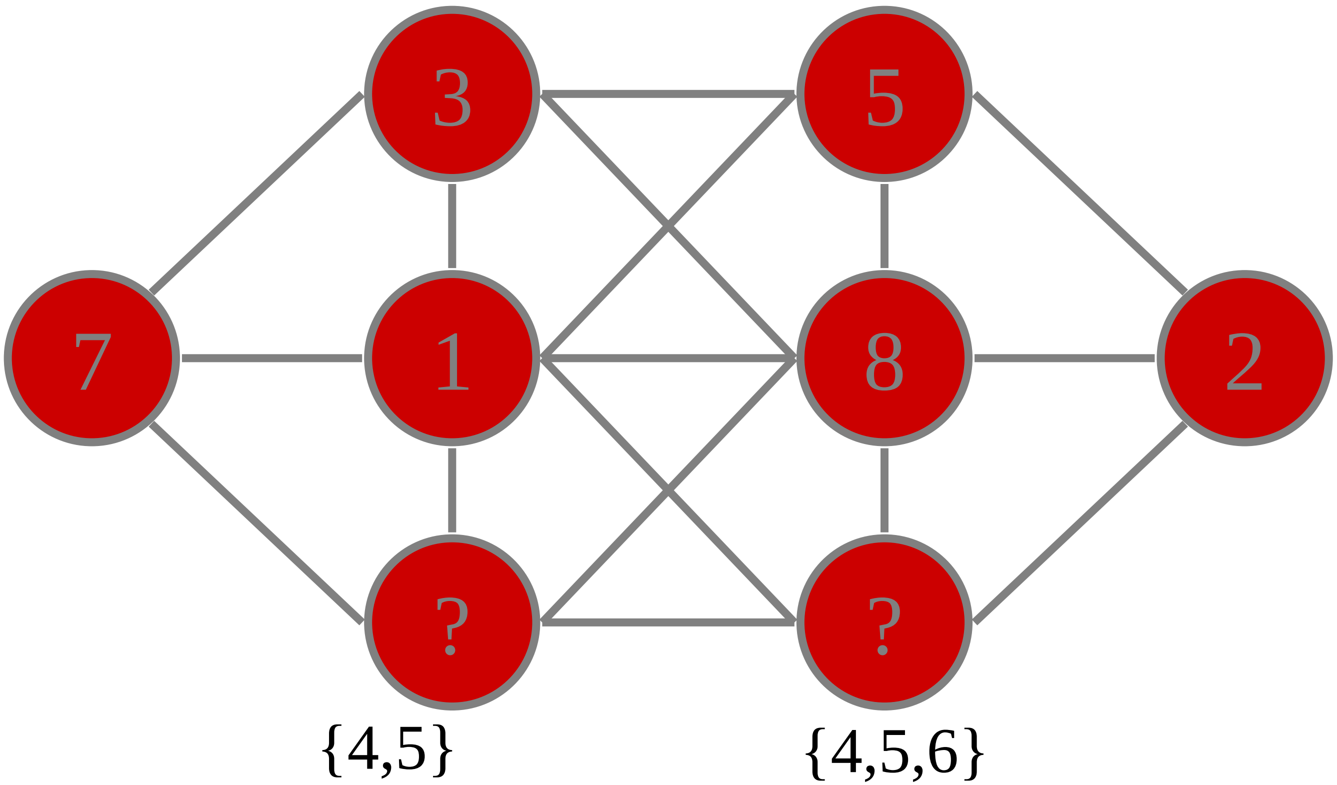
Guess another value …

Inference/propagation



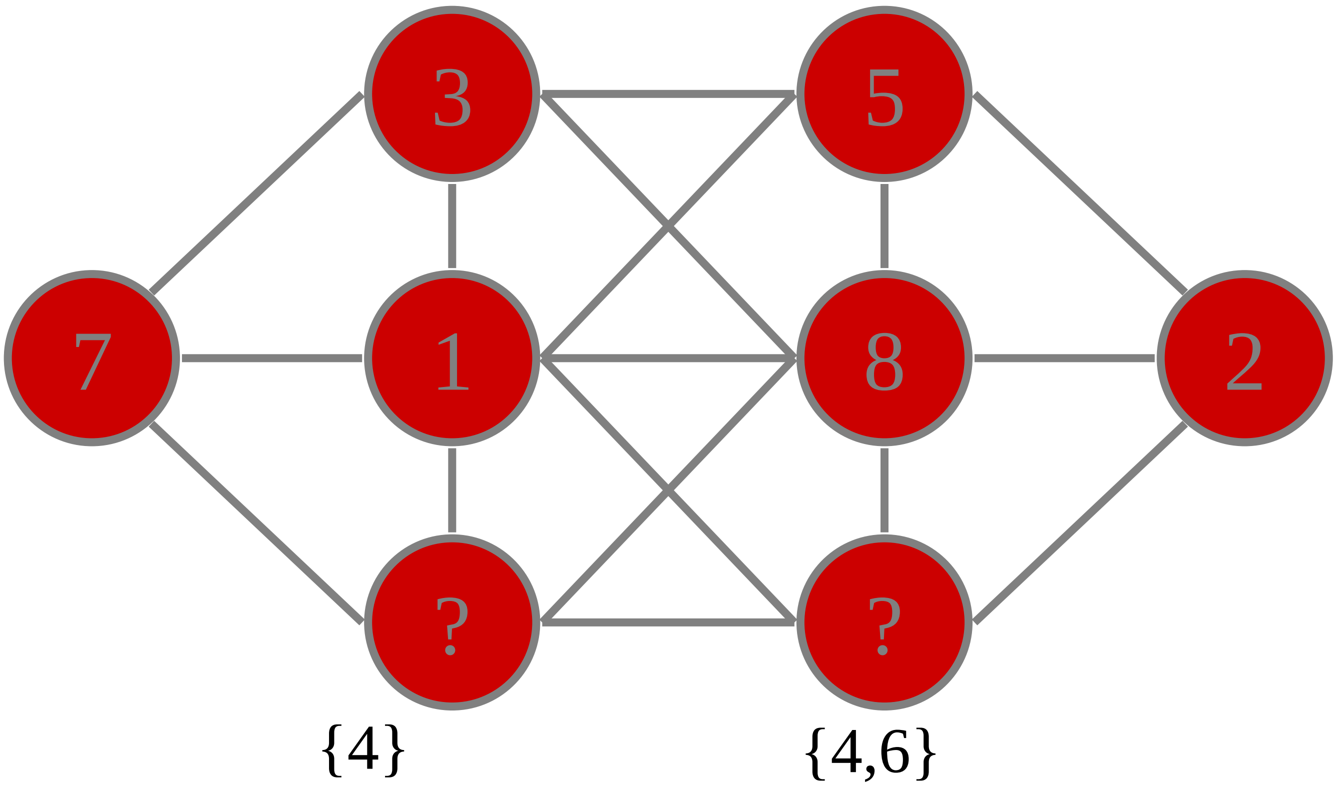
Guess another value …

Inference/propagation



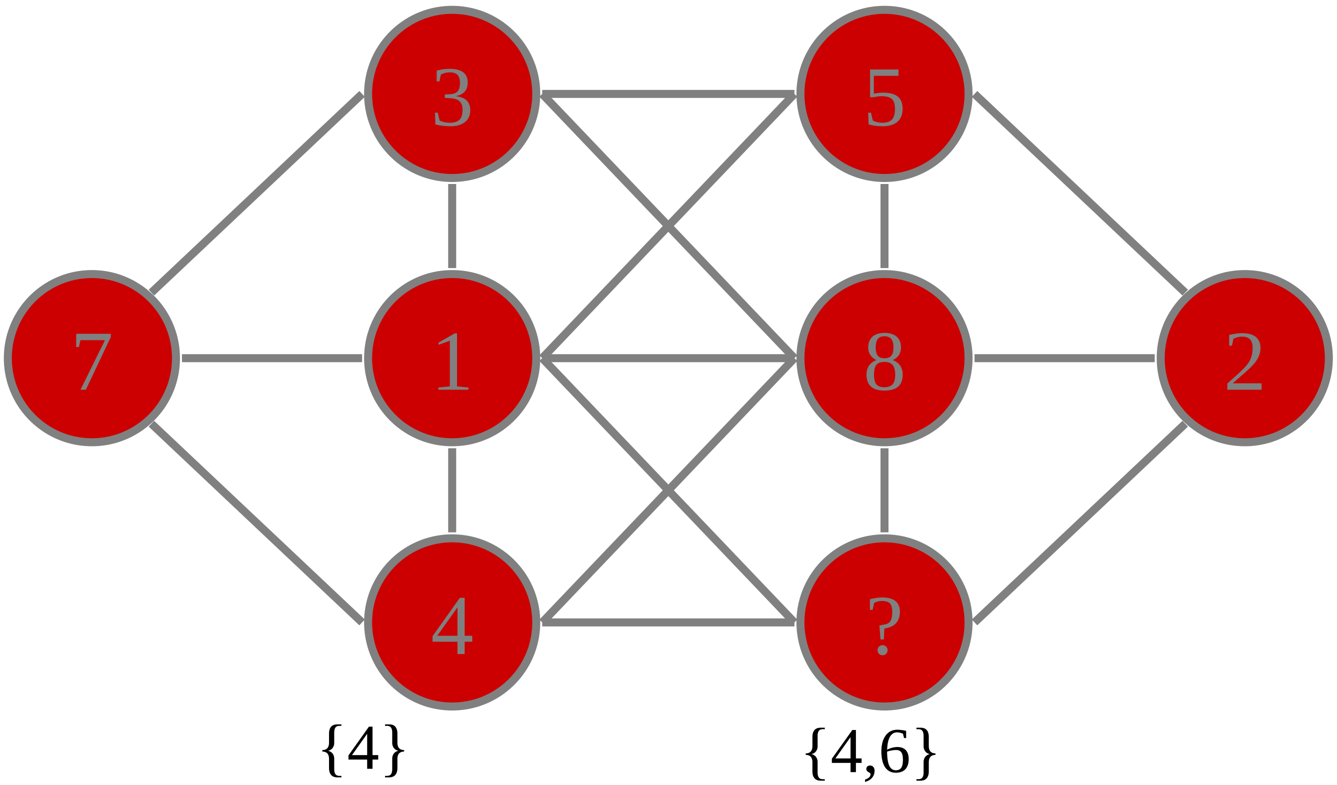
And propagate …

Inference/propagation



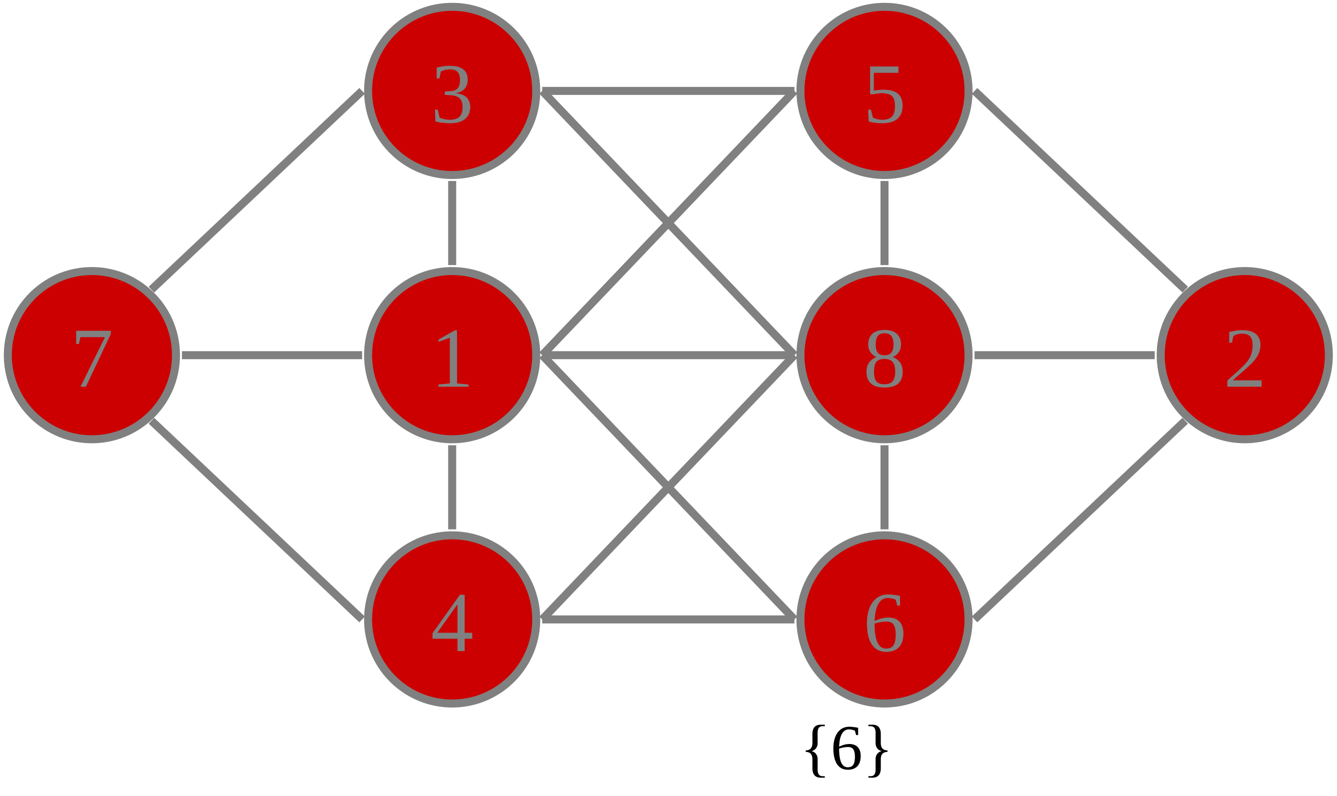
And propagate …

Inference/propagation

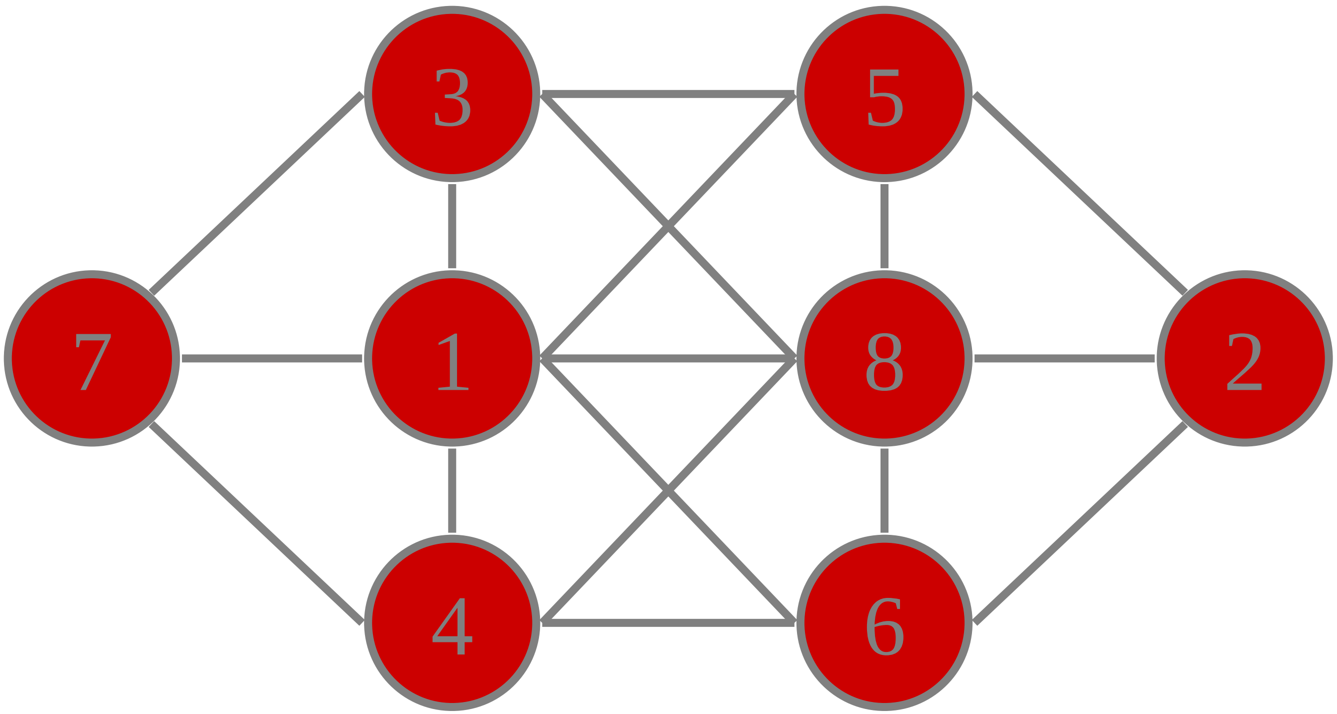


One node has only a single value left …

Inference/propagation



Solution



THE CONVERSATION
Academic rigour, journalistic flair

What problems will AI solve in future? An old British gameshow can help explain

November 3, 2015 1.17pm GMT

Authors

Ian Miguel
Professor of Computer Science, University of St Andrews

Patrick Prosser
Senior Lecturer in Computer Science, University of Glasgow

Disclosure statement

Ian receives research funding from the EPSRC and the Royal Academy of Engineering. He is Director of the Graduate Academy of the Scottish Informatics and Computer Science Alliance and on the board of the Data Lab innovation centre.

Patrick Prosser does not work for, consult, own shares in or receive funding from any company or organisation that would benefit from this article, and has disclosed no relevant affiliations beyond their academic appointment.

Partners

University of Glasgow

No Consecutives

SERIES FOUR

MENTAL

2.00

AUTOMATIC LOCK-IN

Description

A trivial game that looked out of place at this stage of the show. Eight numbers or letters had to be placed onto the grid, so that no consecutive numbers or letters neighboured each other.

Plays

Show	Contestant	Time	Cell	Outcome
Show 4 - 9	Ilan Josephs	2:00	5	Lose
Show 4 - 12	Rob Neasham	2:00	5	Lose
Show 4 - 7	Clare Sardari	2:00	5	Lose
Show 4 - 11	Sandra Finnimore	2:00	5	Lose
Plays...4				**Wins...**0 **Loses...**4 **Lockins...**0 **Win Percentage...**0.0

Crystal Maze problem - Message (HTML)

From: James Trimble (student)

Sent: Wed 18/12/2019 14:36

To: Patrick Prosser; Ciaran Mccreesh

Cc:

Subject: Crystal Maze problem

Patrick,

That's a fun history!

You've probably seen this page already: http://crystalmaze.marcgerrish.com/games/noconsecutives.htm . The puzzle was played four times on crystal maze, and nobody managed to solve it.

It occurred to me that the problem is equivalent to Hamiltonian Path on the complement graph, so it is NP-complete.

James

See more about: James Trimble.

The Core of Constraint Computation

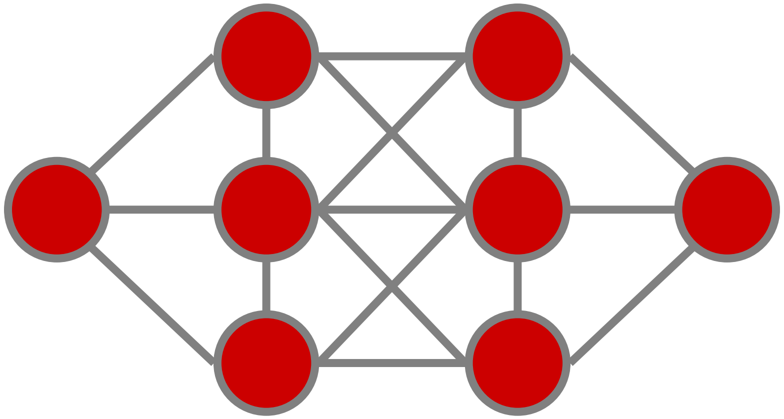
- Modelling
  - Deciding on variables/domains/constraints
- Heuristic Search
- Inference/Propagation
- Symmetry
- Backtracking

A Commercial Reality

- First-tier software vendors use CP technology

Hardness

- The puzzle is actually a hard problem
  - NP-complete



Constraint programming

- Model problem by specifying constraints on acceptable solutions
  - define variables and domains
  - post constraints on these variables
- Solve model
  - choose algorithm
    - incremental assignment / backtracking search
    - complete assignments / stochastic search
  - design heuristics

Constraint satisfaction

- Constraint satisfaction problem (CSP) is a triple <V,D,C> where:
  - V is set of variables
  - Each X in V has set of values, D_X
    - Usually assume finite domain
    - {true,false}, {red,blue,green}, [0,10], …
  - C is set of constraints

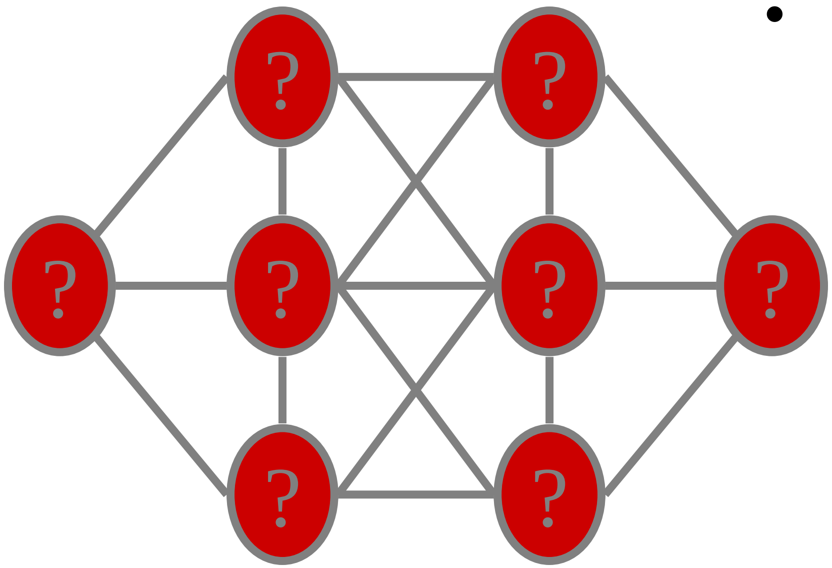
Goal: find assignment of values to variables to satisfy all the constraints

How complex?

Assume

- n variables
- each with a domian size of m
- how many states might we consider?

Example CSP



- Variable, v_i for each node
- Domain of {1, …, 8}
- Constraints
  - All values used

allDifferent(v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8)

  - No consecutive numbers for adjoining nodes

$|v_1 - v_2| > 1$

$|v_1 - v_3| > 1$

…

Constraints

- Constraints are tuples <S,R> where
  - S is the scope, [X1,X2, … Xm]
    - list of variables to which constraint applies
  - R is relation specifying allowed values (goods)
    - Subset of D_X1 x D_X2 x … x D_Xm
    - May be specified intensionally or extensionally

Constraints

- Extensional specification
  - List of goods (or for tight constraints, nogoods)
- Intensional specification
  - X1 =/= X2
  - 5*X1 + 6*X2 < X3
  - alldifferent([X1,X2,X3,X4]), …

more examples?

Do you know any constraint satisfaction problems?

To a man with a hammer, everything looks like a nail.

Sudoku

From Wikipedia, the free encyclopedia

Not to be confused with Sodoku.

Sudoku (数独 *sūdoku*?, listen (help·info)) (English pronunciation: /suːˈdoʊkuː/) is a logic-based,[1][2] combinatorial[3] number-placement puzzle. The objective is to fill a 9×9 grid so that each column, each row, and each of the nine 3×3 boxes (also called blocks or regions) contains the digits from 1 to 9 only one time each. The puzzle setter provides a partially completed grid. Completed puzzles are usually a type of Latin square with an additional constraint on the contents of individual regions.

Sudoku was popularized in 1986 by the Japanese puzzle company Nikoli, under the name Sudoku, meaning *single number*.[4] It became an international hit in 2005.[5]

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

A Sudoku puzzle...

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5

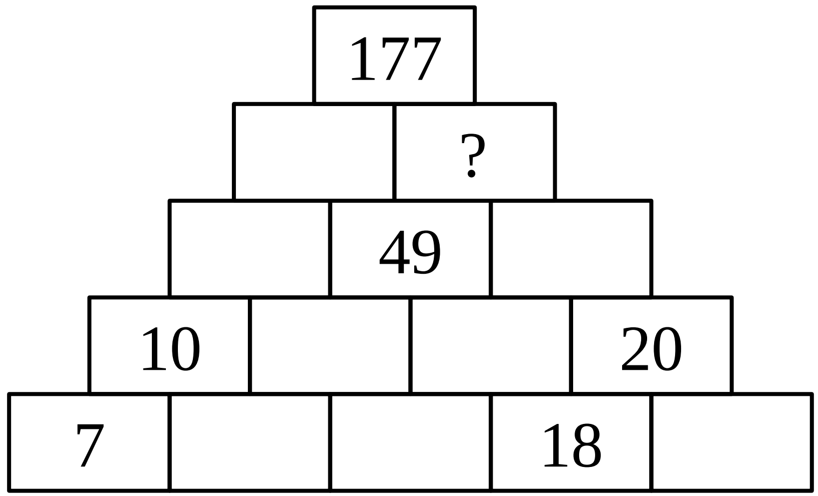
Contents [hide]

History

[edit]

Number puzzles first appeared in newspapers in the late 19th century, when French puzzle setters began experimenting with removing numbers from magic squares. *Le Siècle*, a Paris-based daily, published a partially completed 9×9 magic square with 3×3 sub-squares on November 19, 1892.[6] It was not a Sudoku because it contained double-digit numbers and required arithmetic rather than logic to solve, but it shared key characteristics: each row, column and sub-square added up to the same number.

Scotsman 4/12/2003



In the pyramid above, two adjacent bricks added together give the value of the brick above. Find the value for the brick marked ?

Exam timetabling

An Example, Exam Timetabling

- Someone timetables the exams
- We have a number of courses to examine
    - how many?
        - Dept has 36
        - Faculty?
        - University?
- There are constraints
    - if a student S takes courses Cx and Cy
        - Cx and Cy cannot be at same time!
    - If Cy and Cz have no students in common
        - they can go in room R1 if there is space
    - Temporal and resource constraints

An Example, Exam Timetabling

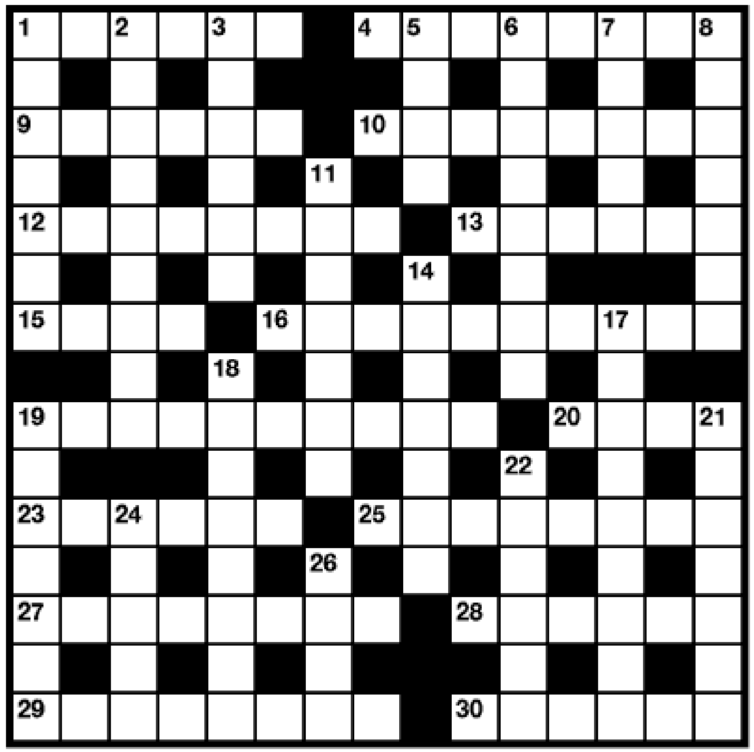
- Represent as graph colouring
  - vertices are courses
  - colours are time
  - vertices have weight (room requirements)
  - edge connects vertices of diff colour
- How complex is this
  - if we have n vertices and k times
    - an n-digit number to the base k?
- How would you solve this
  - backtracking search?
  - Greedy?
  - Something else
    - GA?
    - SA, TS, GLS, HC, ...

An Example, Exam Timetabling

- How does the person solve this?
- Is that person intelligent?
- Is there always a solution?
- If there isn't, do we want to know why?
    - Do you think they can work out "why"?

Crossword puzzle generation

heraldscotland



Monday, 28 Sep 2009

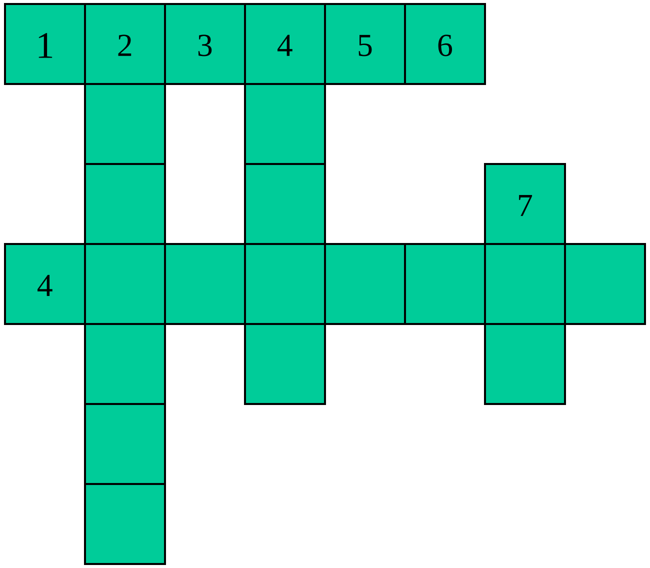
Across

1 Mass-produced legislation? [3,3]
4 Guard ordered to breached line [8]

Down

1 Not subject put on in advance [7]
2 Club about to give female support [9]

An example

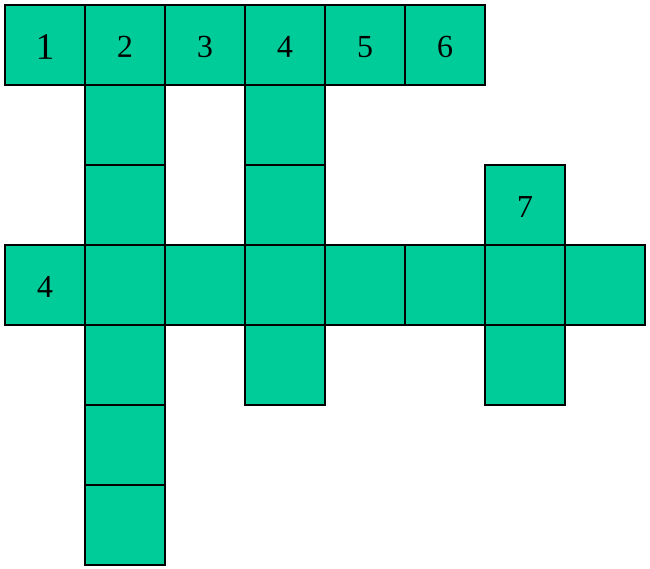


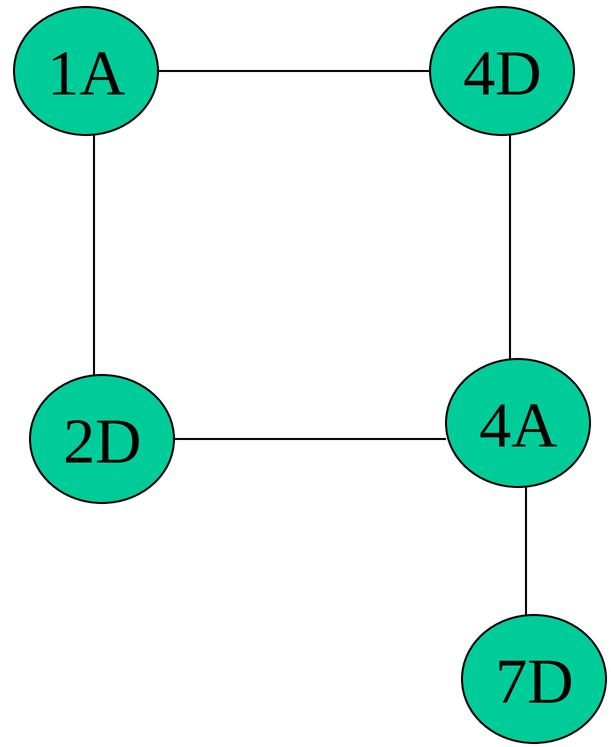
Make a crossword puzzle!

Given the above grid and a dictionary, fill it.

Then go get the clues (not my problem)

An example

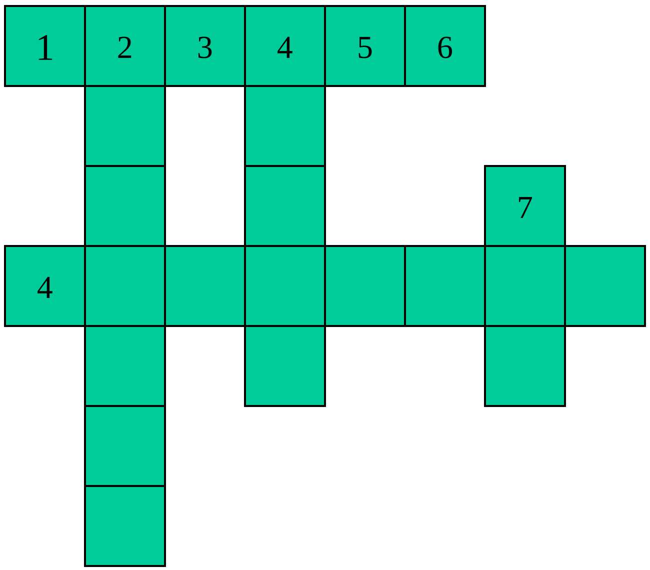


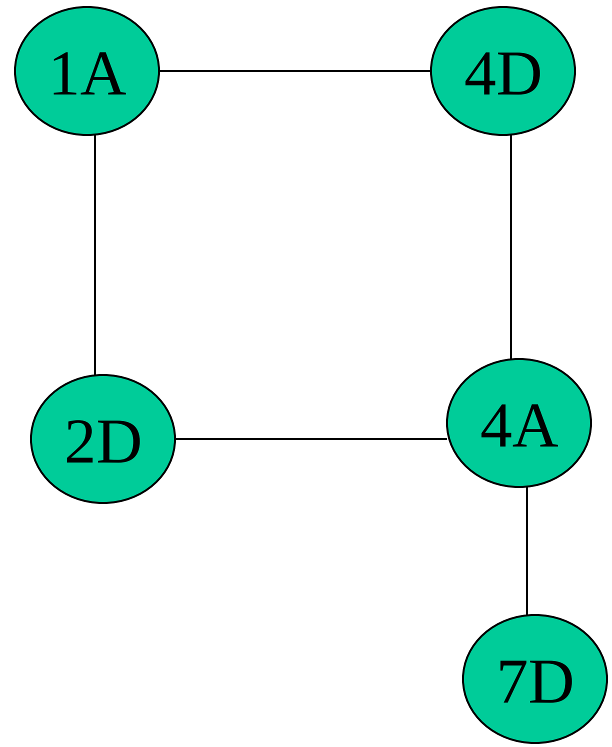


1A	1 across
4D	4 down
2D	2 down
4A	4 across
7D	7 down

Variables

An example





1A-4D: 4th of 1A equals 1st of 4D

1A-2D: 2nd of 1A equals 1st of 2D

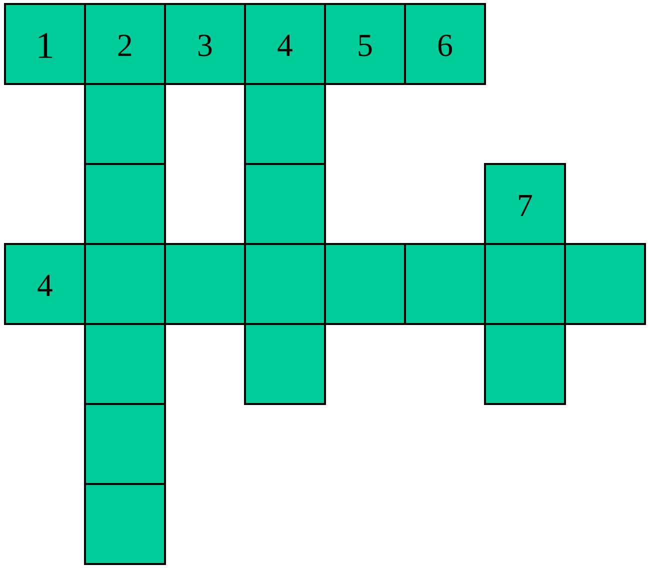
2D-4A: 4th of 2D equals 2nd of4D

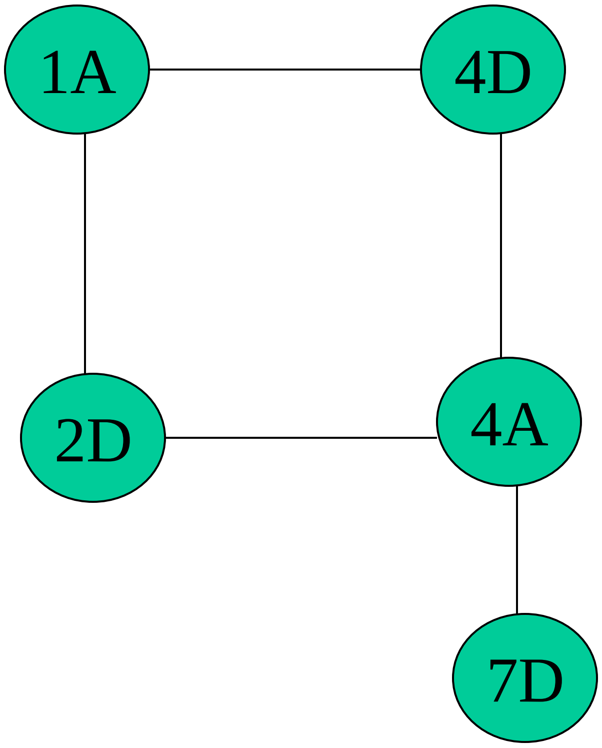
4D-4A: 4th of 4A equals 4th of 4D

4A-7D: 7th of 4A equals 2nd of 7D

Constraints

An example





1A: any 6 letter word

4A: any 8 letter word

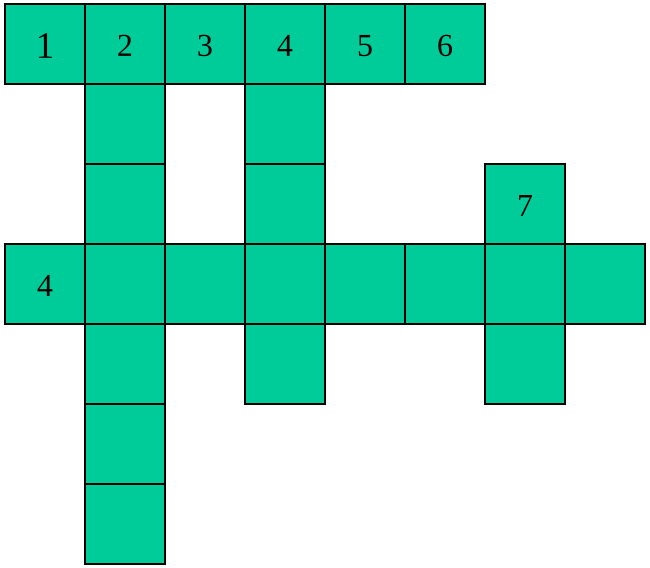
4D: any 5 letter word

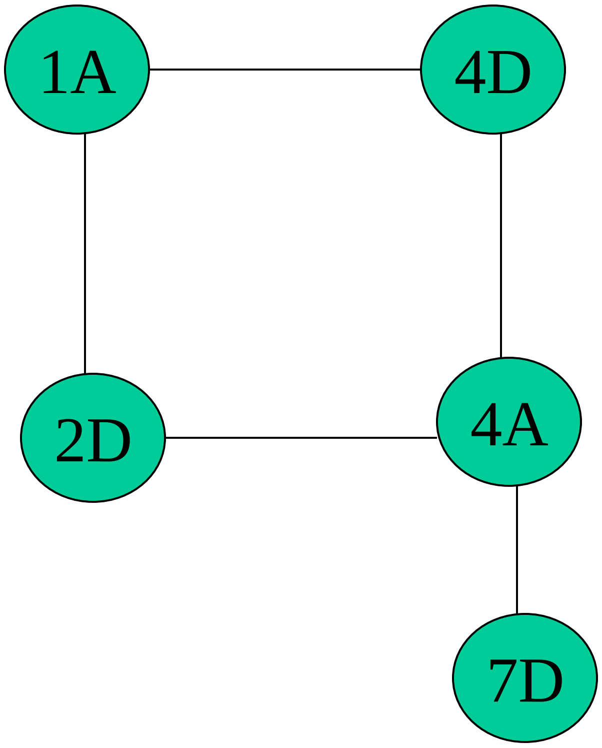
2D: any 7 letter word

7D: any 3 letter word

Domains (also unary constraints!)

An example

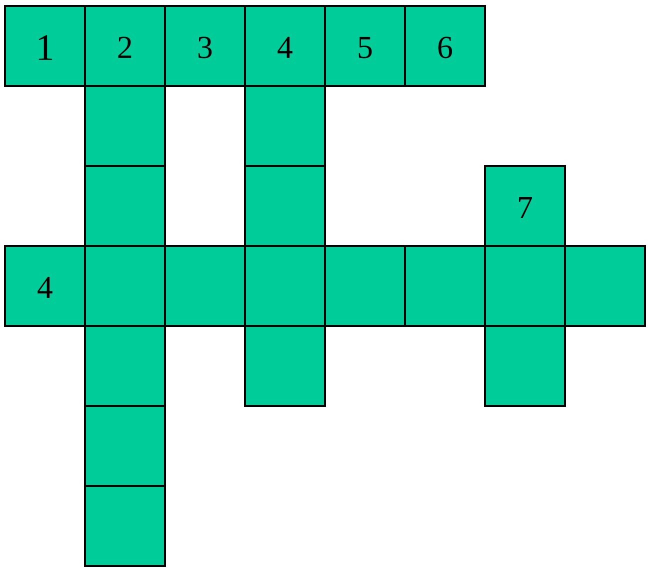


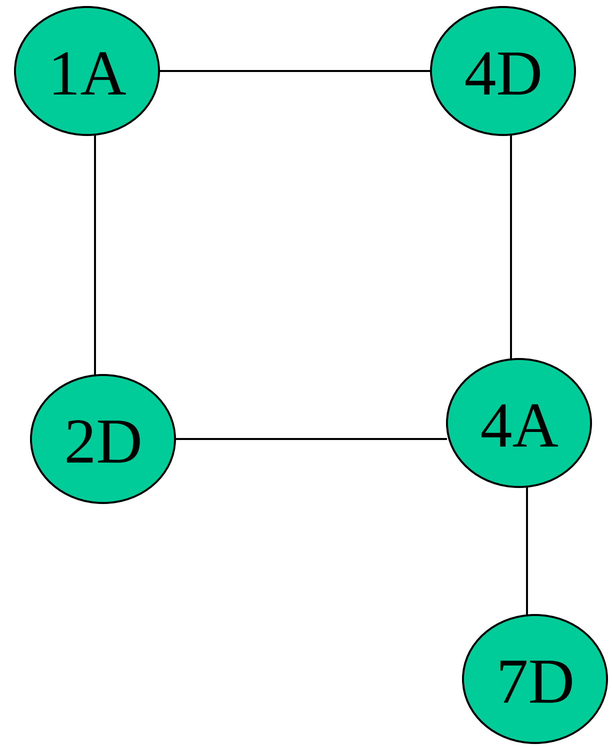


Find an assignment of values to variables, from their domains, such that the constraints are satisfied (or show that no assignment exists)

A CSP!

An example





Choose a variable

Assign it a value

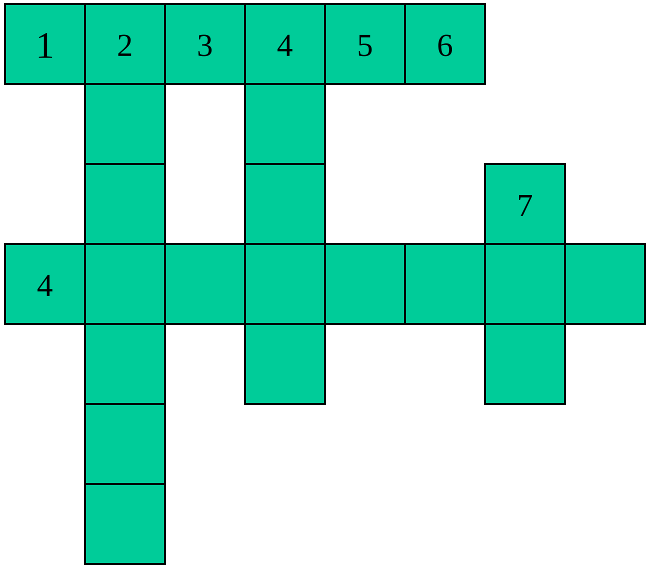
Check compatibility

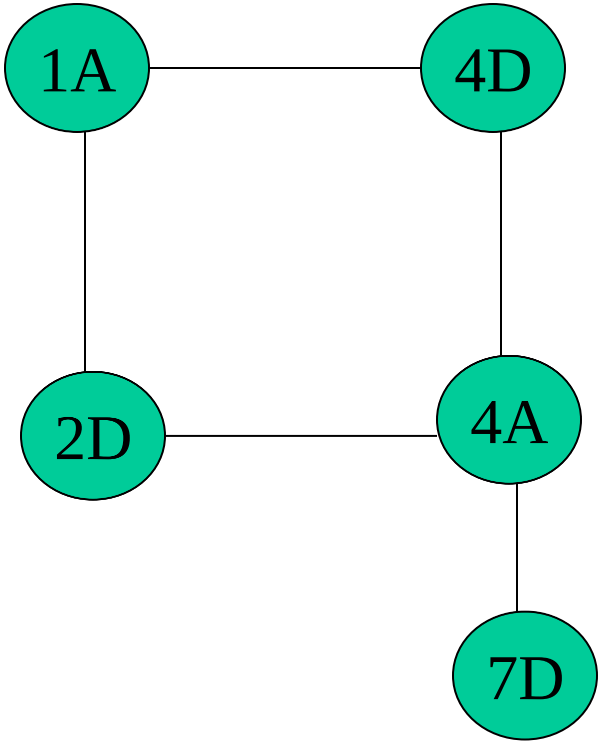
If not compatible try a new value

If no values remain re-assign previous variable

Good old fashioned BT!

Questions?





What variable should I choose?

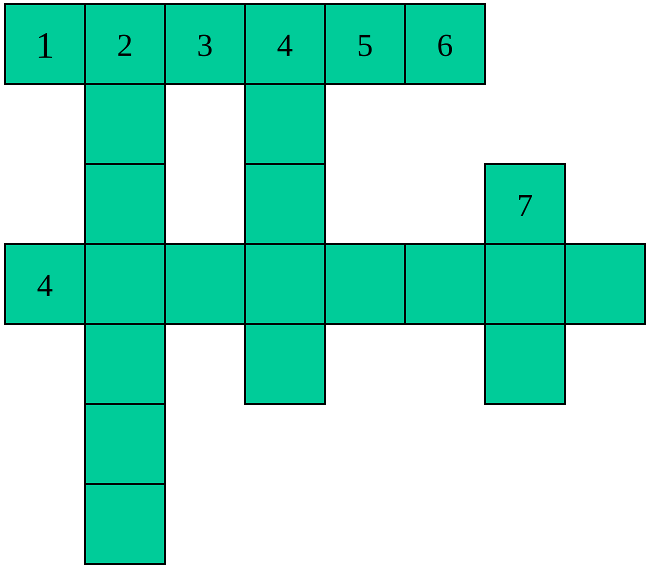
What value should I choose?

What reasoning can I do when making an assignment?

What reasoning can I do on a dead end?

Decisions, decisions!

An example



Is there an alternative representation?

Problems of interest to CP

These are some of the problems that have been tackled by CP

- factory scheduling (JSSP)
- vehicle routing (VRP)
- packing problems (NumPart and BinPack)
- timetabling (exams, lectures, trains)
- configuration and design (hardware)
- workforce management (call centres, etc)
- car sequencing (assembly line scheduling)
- supertree construction (bioinformatics)
- network design (telecoms problem)
- gate arrival (at airports)
- logistics (Desert Storm an example)
- aircraft maintenance schedules
- aircraft crew scheduling (commercial airlines)
- air cover for naval fleet

What will be covered in course

- the technology behind constraint programming (cp)
- cp in JChoco/java
- modelling and solving problems
- the state of the art