

Put a different number in each circle (1 to 8) such that adjacent circles cannot take consecutive numbers Constraint Programming An Introduction by example

with help from Toby Walsh, Chris Beck, Barbara Smith, Peter van Beek, Edward Tsang, ...

A Puzzle

- Place numbers 1 through 8 on nodes
 - Each number appears exactly once



Which nodes are hardest to number?





Which are the least constraining values to use?



Values 1 and 8



Values 1 and 8



Symmetry means we don't need to consider: 8 1



We can now eliminate many values for other nodes









By symmetry









By symmetry





Value 2 and 7 are left in just one variable domain each









Guess a value, but be prepared to backtrack ...



Guess a value, but be prepared to backtrack ...







Guess another value ...



Guess another value ...







One node has only a single value left ...



Solution





What problems will AI solve in future? An old British gameshow can help explain

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Partners



000



A trivial game that looked out of place at this stage of the show. Eight numbers or letters had to be placed onto the grid, so that no consecutive numbers or letters neighboured each other.

Plays						
Show	Contestant	Time	Cell	Outcome		
Show 4 - 9	Ilan Josephs	2:00	5	Lose		
<u>Show 4 - 12</u>	Rob Neasham	2:00	5	Lose		
<u>Show 4 - 7</u>	Clare Sardari	2:00	5	Lose		
<u>Show 4 - 11</u>	Sandra Finnimore	2:00	5	Lose		
Plays4				Wins0 Loses4 Lockins0 Win Percentage0.0		
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From: James Trimble (student) Sent: Wed 18/12/2019 14:36 To: Patrick Prosser; Ciaran Mccreesh Cc: Cc:								
Subject: Crystal Maze problem Patrick, That's a fun history! You've probably seen this page already: http://crystalmaze.marcgerrish.com/games/noconsecutives.htm . The puzzle was played four times on crystal maze, and nobody managed to solve it. It occurred to me that the problem is equivalent to Hamiltonian Path on the complement graph, so it is NP-complete. James								

The Core of Constraint Computation

- Modelling
 - Deciding on variables/domains/constraints
- Heuristic Search
- Inference/Propagation
- Symmetry
- Backtracking

A Commercial Reality

• First-tier software vendors use CP technology



Hardness

• The puzzle is actually a hard problem - NP-complete



Constraint programming

- Model problem by specifying constraints on acceptable solutions
 - define variables and domains
 - post constraints on these variables
- Solve model
 - choose algorithm
 - incremental assignment / backtracking search
 - complete assignments / stochastic search
 - design heuristics

Constraint satisfaction

- Constraint satisfaction problem (CSP) is a triple <V,D,C> where:
 - V is set of variables
 - Each X in V has set of values, D_X
 - Usually assume finite domain
 - {true,false}, {red,blue,green}, [0,10], ...
 - C is set of constraints

Goal: find assignment of values to variables to satisfy all the constraints

How complex?

Assume

- n variables
- each with a domian size of m
- how many states might we consider?

Example CSP

- Variable, v_i for each node
- Domain of {1, ..., 8}
- Constraints

- All values used allDifferent($v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8$)

 No consecutive numbers for adjoining nodes

$$\begin{aligned} |\mathbf{v}_1 - \mathbf{v}_2| &> 1 \\ |\mathbf{v}_1 - \mathbf{v}_3| &> 1 \end{aligned}$$



Constraints

- Constraints are tuples <S,R> where
 - S is the scope, [X1,X2, ... Xm]
 - list of variables to which constraint applies
 - R is relation specifying allowed values (goods)
 - Subset of D_X1 x D_X2 x ... x D_Xm
 - May be specified intensionally or extensionally

Constraints

- Extensional specification
 List of goods (or for tight constraints, nogoods)
- Intensional specification
 - X1 =/= X2
 - -5*X1 + 6*X2 < X3
 - alldifferent([X1,X2,X3,X4]), ...

more examples?

Do you know any constraint satisfaction problems?

To a man with a hammer, everything looks like a nail.

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navigation Main page Contents Featured content Current events Random article Standom article Search Go Search Contents (hidg) Abuoku (%3% südoku¹, 4) listen (help-into) (English prenundiation: /Su¹dOuku¹) is a logic-based, full combinatorial^[3] number-placement puzzle. The objective is to fill a 9-9 grid so that each of the nine 3×3 boxes (also called blocks or regions) contains the digits from 1 to 9 only one time each. The puzzle setter provides a partially completed grid. Completed puzzles are usually a type of Latin square with an additional constraint on the contents of individual regions. Sudoku was popularized in 1986 by the Japanese puzzle company Nikoli, under the name Sudoku. meaning single number.^[4] It became an international hit in 2005.^[5] Contents (hidg) 1 History 2 Variants Contents (hidg) 2 Variants Contents (hidg) 2 Variants B Computive potal Computive potal Company (Nikoli, under the name Sudoku, meening single number.^[4] It became an international hit in 2005.^[5] Contents (hidg) 2 Variants Contents (hidg) 2 Variants B Extent changes Contents (hidg) 2 Variants B Extent links S S S 7 6 1 1 4 2 3 S S S 7 6 1 1 4 2 3 S S S 7 6 1 1 4 2 3 S S S 7 6 1 1 4 2 3 S S S 7 6 1 1 4 2 3 S S S 7 7 6 1 1 4 2 3 S S S 7 7 6 1 1 4 2 3<!--</td--><td>WIKIPEDIA The Free Encyclopedia</td><td>From Wikipedia, the free er</td><td>ncyclopedia</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>-</td>	WIKIPEDIA The Free Encyclopedia	From Wikipedia, the free er	ncyclopedia										-
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Scotsman 4/12/2003



In the pyramid above, two adjacent bricks added together give the value of the brick above. Find the value for the brick marked?

Exam timetabling

An Example, Exam Timetabling

- Someone timetables the exams
- We have a number of courses to examine
 - how many?
 - Dept has 36
 - Faculty?
 - University?
- There are constraints
 - if a student S takes courses Cx and Cy
 - Cx and Cy cannot be at same time!
 - If Cy and Cz have no students in common
 - they can go in room R1 if there is space
 - Temporal and resource constraints

An Example, Exam Timetabling

- Represent as graph colouring
 - vertices are courses
 - colours are time
 - vertices have weight (room requirements)
 - edge connects vertices of diff colour
- How complex is this
 - if we have n vertices and k times
 - an n-digit number to the base k?
- How would you solve this
 - backtracking search?
 - Greedy?
 - Something else
 - GA?
 - SA, TS, GLS, HC, ...

An Example, Exam Timetabling

- How does the person solve this?
- Is that person intelligent?
- Is there always a solution?
- If there isn't, do we want to know why?
 - Do you think they can work out "why"?

Crossword puzzle generation





Make a crossword puzzle!

Given the above grid and a dictionary, fill it.

Then go get the clues (not my problem)





1A	1 across
4D	4 down
2D	2 down
4A	4 across
7D	7 down

Variables





1A-4D: 4th of 1A equals 1st of 4D
1A-2D: 2nd of 1A equals 1st of 2D
2D-4A: 4th of 2D equals 2nd of 4D
4D-4A: 4th of 4A equals 4th of 4D
4A-7D: 7th of 4A equals 2nd of 7D





1A: any 6 letter word
4A: any 8 letter word
4D: any 5 letter word
2D: any 7 letter word
7D: any 3 letter word

Domains (also unary constraints!)





Find an assignment of values to variables, from their domains, such that the constraints are satisfied (or show that no assignment exists)

A CSP!



Choose a variable Assign it a value Check compatibility If not compatible try a new value If no values remain re-assign previous variable



Good old fashioned BT!

Questions?



What reasoning can I do on a dead end?

Decisions, decisions!

7D

4D

4A



Is there an alternative representation?

Problems of interest to CP

These are some of the problems that have been tackled by CP

- factory scheduling (JSSP)
- vehicle routing (VRP)
- packing problems (NumPart and BinPack)
- timetabling (exams, lectures, trains)
- configuration and design (hardware)
- workforce management (call centres, etc)
- car sequencing (assembly line scheduling)
- supertree construction (bioinformatics)
- network design (telecoms problem)
- gate arrival (at airports)
- logistics (Desert Storm an example)
- aircraft maintenance schedules
- aircraft crew scheduling (commercial airlines)
- air cover for naval fleet

- the technology behind constraint programming (cp)
- cp in JChoco/java
- modelling and solving problems
- the state of the art

