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Information-theoretic characterization of uncertainty in manual control

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Abstract—We present a novel approach for quantifying the impact of uncertainty in manual control, based on information and control theories and utilizing the information-theoretic capacity of *empowerment*, a task-independent universal utility measure. *Empowerment* measures, for agent-environment systems with stochastic transitions, how much influence, which can be sensed by the agent sensors, an agent has on its environment. It enables combining different types of disturbances, arising in human-machine systems (i.e. noise, delays, errors, etc.), into one single measure. We expand *empowerment* to manual control, demonstrate its application in the field of HCI and evaluate it in a user study. Results showed that *empowerment* is strictly monotonic in relation to the means of standard performance metrics total time off-target, perceived uncertainty, perceived performance and frustration, which suggests its potential in making theoretical predictions of other measures. Loss of *empowerment* implicated interesting trends in activity levels, which open a new area for future work. Results suggest the potential *empowerment* has in providing better theoretical foundations for the science of HCI.

Index Terms—Empowerment, Uncertainty, Human-Computer Interaction, Human Factors, Information theory, Manual control.

I. INTRODUCTION

A system's usability depends on various sources of uncertainty, which have an impact on the design process. Transmission delays and measurement noise create uncertainty about the actual current state of a human–machine system. The longer the delays and higher the noise, the higher the uncertainty. This imposes decisions on designers when trying to optimize system's performance. Switching between different types of networks and peaks in the traffic create variability in transmission delays. Systems have processing bottlenecks creating varying delays in different processes. These disturbances are largely unpredictable from human perspective and create the perception of slow and unresponsive devices or applications. Uncertainty poses specific risks and requires detailed sensitivity analysis, for which we need proper tools and measures. The quality of control depends on feedback that must reflect the uncertainty of system beliefs. Interfaces should be honest and work with the uncertainty, not just filter it out [10]. Appropriate use of uncertain feedback can lead to smoother interaction, with user behaviour regularised appropriately [7], [8].

Lag is recognized as a major bottleneck for usability [9]. To compensate for varying delays in current networks designers need to optimize systems for speed, reliability and overall user experience. There is a trade-off between these factors, since if we would only optimize for speed, the system would behave erratically as delay varies or it would become sluggish if we artificially increased the inherent delay for the sake of stability. Analyzing this trade-off is not a trivial task and requires proper tools. We have objective measures for human performance, as success rate or completion time and subjective ones, as user experience questionnaires (e.g. NASA-TLX [2], a multi-dimensional workload assessment tool). These tools, however, usually require extensive studies, which come at a price and still pose certain risks regardless of the evaluated point density.

To evaluate system's usability in various environmental conditions we could take the approach of *empowerment* [5] as an objective measure reflecting how much control the user has in the course of interaction. *Empowerment* could provide a good analytical tool for performance tuning by revealing critical salient points in the system[1] design while avoiding brute-force testing. Analyzing the trends and the gradients on the empowerment curve could give direct insight into the underlying properties and provide confidence regions for the system's parameters. These insights will help designers to make a better choice for systems to evaluate. Using the *empowerment* measure as a first step in the system's analysis will improve quality of design, and at the same time reduce risk and evaluation costs.

II. BACKGROUND

Our view on the fundamentals of interaction is that users' behaviour is about them controlling their perceptions [11]. The more control they have over their perceptions the more empowered they are by the human-machine interface to achieve their goals. Conversely, they are less empowered if they cannot perceive the effect of their decisions.

The term *empowerment* was introduced in its technical sense by [6], as an information-theoretic 'universal utility', representing the channel capacity between an agent's actions and its sensory observations in subsequent time steps. *Empowerment* is defined for stochastic dynamic systems, where transitions arise as the result of making a decision, such as an agent interacting with an environment. It is based on the perception-action loop of the agent-environment coupling unrolled over time and is fully specified by the dynamics (i.e. transition probabilities). It captures the amount of information

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[1]For systems where variability is reasonably well understood.

that can be injected by an agent into its environment and then perceived by its sensors, and is defined as the channel capacity from the sequence of actions $A_t, A_{t+1}, ..., A_{t+n-1}$ to the perceptions S_{t+n} after an arbitrary number of time steps

$$C(A_t, ..., A_{t+n-1} \to S_{t+n}) = \sup_{p(\vec{a})} I(A_t, ..., A_{t+n-1}; S_{t+n})$$

with $p(\vec{a})$ being the probability distribution function of the action sequences ($\vec{a} = (a_t, ..., a_{t+n-1})$). For simplicity and without loss of generality, we will consider single actions, which however can consist of action sequences as well.

We assume discrete state ($\mathcal{X} \subset \mathbb{Z}$) and action ($\mathcal{A} = \{1, ..., N_A\}$) spaces. The transition function is given in terms of a density $p(x_{t+1}|x_t, a_t)$, which denotes the probability of going from state x_t to x_{t+1} when making decision a_t. Let $\mathcal{X}'$ denote the random variable associated with x' given x and let the choice of a particular action a be modelled by random variable $\mathcal{A}'$. The *empowerment* $\mathfrak{E}(x)$ in state x (given the transition density $p(x'|x, a_\nu)$) is then defined as the Shannon channel capacity between $\mathcal{A}'$, the choice of an action sequence, and $\mathcal{X}'$, the resulting successor state ($\mathfrak{E} : \mathcal{X} \to \mathbb{R}^{\geq 0}$)

$$\mathfrak{E}(x) = \max_{p(a)} I(\mathcal{X}'; \mathcal{A}'|x) = \max_{p(a)}\{H(\mathcal{X}'|x) - H(\mathcal{X}'|\mathcal{A}', x)\}.$$

The mutual information maximization is with respect to all possible distributions $p(a)$ over $\mathcal{A}$. The entropies are given by

$$H(\mathcal{X}'|x) = -\sum_{x' \in \mathcal{X}} p(x'|x) \log p(x'|x),$$

$$H(\mathcal{X}'|\mathcal{A}', x) = -\sum_{\nu=1}^{N_A} p(a_\nu) H(\mathcal{X}'|\mathcal{A}' = a_\nu, x)$$

$$= -\sum_{\nu=1}^{N_A} p(a_\nu) \sum_{x' \in \mathcal{X}} p(x'|x, a_\nu) \log p(x'|x, a_\nu). \quad (1)$$

For a simple example[2] in the context of HCI consider a pop-up window with 'OK' and 'Cancel' buttons and the uniform distribution of $p(a)$ over $\mathcal{A}$. In the deterministic case when both actions have distinct outcomes $p(x_1|x, a_1)=p(x_2|x, a_2)=1$, using $p(x'|x)=\sum_{i=1}^{N_A} p(x'|x, a_i)p(a_i)$, we get $H(\mathcal{X}'|x)=\log(2)$ and $H(\mathcal{X}'|\mathcal{A}', x)=0$, hence $I(\mathcal{X}'; \mathcal{A}'|x)=log(2)\approx0.69$, which is also the maximum ($\mathfrak{E}(x)$). If we add uncertainty to the pdf, e.g. $p(x_1|x, a_2)=0.1$ and $p(x_2|x, a_2)=0.9$, we get $H(\mathcal{X}'|x)\approx0.69$, $H(\mathcal{X}'|\mathcal{A}', x)\approx0.16$ and $I(\mathcal{X}'; \mathcal{A}'|x)$ decreases to 0.53. If we increase uncertainty to the maximum $p(x_1|x, a_1)=p(x_2|x, a_2)=p(x_1|x, a_2)=p(x_2|x, a_1)=1/2$ we get $H(\mathcal{X}'|x)=H(\mathcal{X}'|\mathcal{A}', x)=\log(1/2)$ or $I(\mathcal{X}'; \mathcal{A}'|x)=0$, again a maximum meaning zero empowerment – here we still have a binary option to make a decision, but we cannot perceive the difference in the outcomes, since both choices lead to the same states as the transition probabilities of the two actions completely overlap. Intuitively, *empowerment* measures the number of actions available to the user on a logarithmic scale, the outcome of which can be perceived. *Empowerment* is zero if, regardless of the action, the outcome will be the same and is maximal if every action has a distinct outcome.

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[2] A more elaborate numerical example can be found in [4].

III. MODEL

Initial work on *empowerment* focused on discrete grid-worlds [5], and later extended to continuous control tasks well known in the area of reinforcement learning [4]. This prior work [4]–[6] aimed to provide a natural utility function with a dense feedback for driving behaviour and investigated specific survival-type scenarios. Our goal is to expand the principle of *empowerment* to manual control[3] and introduce it to HCI research as a novel framework for supporting design.

A. Information theoretic

Since *empowerment* was originally defined as a function of state, which in HCI is often uncertain, we need to replace the exact state in the model by a prediction, the more disturbances affecting the system the more uncertain the prediction.

We will consider a Bayesian network with introduced time delay d between the actuation and the sensing channel, reflecting that the effect of the action a_t will only be perceivable at time $t + d$. We assume the system is fully defined in terms of the 1-step transitions and will be interested in more general d-step interactions where $d \geq 1$ is a random variable from a given distribution. We will consider the sequence $\vec{a}_{t-1} = (a_{t-d}, a_{t-d+1}, ..., a_{t-1})$ of d single-step actions and the probability density $p(x|x_t, \vec{a}_{t-1})$ of making the respective d-step transition. Assuming that we know the complete action history, let $p(\vec{a}_{t-1})$ denote the distribution of the delay d.

Given the latest (delayed) observation x_t and the 1-step ($p(x_{t+1}|x_t, a_t)$) and d-step ($p(x|x_t, \vec{a}_{t-1})$) transitions we define the 1-step transitions from the *uncertain* state $\tilde{x}_t$ as follows

$$p(x_{t+1}|\tilde{x}_t, a_t) = \int_{\mathcal{X}} \sum_{\vec{a}_{t-1}} p(\vec{a}_{t-1})p(x|x_t, \vec{a}_{t-1})p(x_{t+1}|x, a_t)dx,$$
$$\tag{2}$$

for every $x_{t+1} \in \mathcal{X}$ and $a_t \in \mathcal{A}$. The integral in (2) sums up the probability of going from state x_t to x under $\vec{a}_{t-1}$ and then from x to x_{t+1} under a_t. Substituting (2) in (1) let

$$\mathfrak{E}(\tilde{x}_t) = \max_{p(a_t)} \sum_{\nu=1}^{N_A} p(a_{t,\nu}) \int_{\mathcal{X}} p(x_{t+1}|\tilde{x}_t, a_{t,\nu}) \cdot$$

$$\log\left\{\frac{p(x_{t+1}|\tilde{x}_t, a_{t,\nu})}{\sum_{i=1}^{N_A} p(x_{t+1}|\tilde{x}_t, a_{t,i})p(a_{t,i})}\right\} dx_{t+1} \tag{3}$$

define the (1-step) *empowerment* in the *uncertain* state $\tilde{x}_t$.

The proposed model extends the *empowerment* formalism to cases where the exact state is unknown and where the delay varies in time according to a given probability distribution.

B. Control theoretic

We explore two types of disturbances in the feedback loop of our control system: communication delays and Gaussian state noise (Fig. 1). The actions are delayed in time, the system state is corrupted by an additive Gaussian state noise, and the observation is based on a predictive display. The goal is to keep the error within a certain range. We define the human–machine system dynamics following the notation on Fig. 1:

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[3] To simplify issues we will assume mapping to a narrow state.
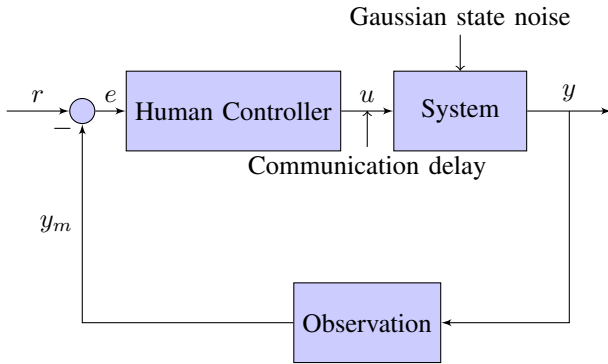
Fig. 1: Control system view of the feedback loop [3].

$$x_t = f(x_{t-1}, h(u_t)); y_t = x_t, \qquad (4)$$

where $f(x, U) = x + U + \mathcal{N}(0, \sigma); h(u_t) = U_t; y_m = g(y_{t-d})$.

The system's transfer function f is a pure displacement perturbed with a Gaussian state noise, the control transfer function h is a mapping of the continuous control to a set of discrete actions and g is the predictive feedback display, which makes a prediction based on the last available observation y_{t-d} (d is the communication delay) and the full control history.

C. Simulations

To explore the properties of our *empowerment* model (3) we performed series of Monte Carlo simulations to create the transition pdf (Fig. 2 presents one sample), in which various levels of constant delays and Gaussian state noise are accounted for in the computation[4] of *empowerment* through our control model (4) with a set of three moving actions {left, neutral, right}. As explained earlier, here *empowerment* converges to $\log(3) = 1.0986$ as uncertainty decreases.

D. Goals

- Explore how user behaviour adapts to uncertainty.
- Determine how users perform in noisy environments.
- Propose a novel theoretical measure for HCI research.

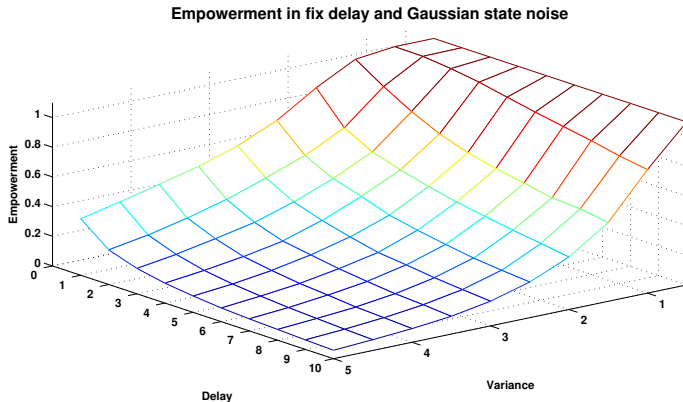[4]We used the Blahut-Arimoto algorithm [1] for the channel capacity (3).

Fig. 2: Empowerment as a function of disturbances in the control loop – constant delays and Gaussian state noise $\mathcal{N}(0, \sigma)$.

Our research questions were:

1) What is the relation of *empowerment* to standard performance metrics?
2) Can *empowerment* bring more insight into human interaction with a given system in uncertain environments?
3) How representative is *empowerment* as a usability measure for a particular system?

IV. EXPERIMENTAL DESIGN

A. Prototype system

Our prototype implements a simple car simulator-like game on Nokia N9 mobile phone, enabling users to steer a car on a randomly moving road, using one-dimensional touch input. The task is a 1-D tracking of a horizontally moving road with added artificial disturbances to the feedback channel and to the control input. The car is steered horizontally with a 3-way joystick on the touch-screen (Fig. 3). The control area is divided into three segments, corresponding to three discrete actions – moving horizontally the car by -5, 0 or 5 pixels. The goal is to maintain the car in the middle of the road and leaving the road is considered as failure. When the car is outside the road a beeping audio cue is played through headphones. The control loop is run at 33Hz, allowing for real-time interaction.

By varying the noise and delay levels we insert different types of disturbances into the system (Table I). To create an uncertain visual display we replaced the car with a single random point, hidden in a cloud, which has different properties in different levels of empowerment. The movement of the point cloud is affected by artificial Gaussian state noise, whereas its size is affected by communication delay. In the delayed case we used a predictive display, propagating the data points in respect to control inputs, which increases the cloud's size.

We designed three different types of feedback loop, representing High, Medium and Low levels of empowerment (Table I) – each type corresponding to one of the conditions tested in this study. The difference between the visual displays in the High and Medium empowerment conditions (Fig. 3, no delay) and the Low empowerment condition (Fig. 5, with delay) is in the spread of the cloud which is wider in the delayed predictive display. The movement of the point cloud is smoother and more predictable in the High empowerment condition (no state noise) and more erratic and unpredictable in the Medium empowerment condition (with state noise).

B. Measures

We defined the following performance measures:

- Average duration on-target – $\overline{\sum_{|e_t| \le \varepsilon} \Delta t}$
- Total time off-target – $\sum_{|e_t| > \varepsilon} \Delta t$

TABLE I: Controlled variables and empowerment levels

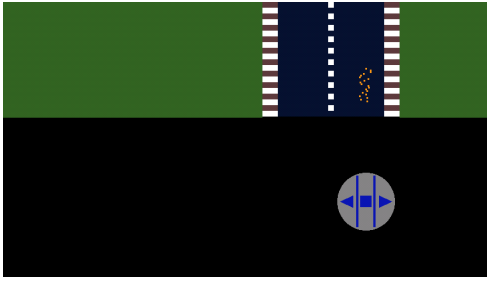| Condition (empowerment)	Delay (steps)	Gaussian state noise $\mathcal{N}(0, \sigma)$ (σ)	Empowerment (nats)
High	0	0	1.0986
Medium	0	5	0.3391
Low	10	5	0.0445

Fig. 3: Visual display (upper half) in High and Medium empowerment (car location is a single point in a narrow point cloud); touch-sensitive control area (gray sphere/lower half).

- Expended finger movement effort – $\Delta u_t = \frac{|u_t - u_{t-1}|}{\Delta t}$
- Resulting change in control input – $\Delta U_t = \frac{|U_t - U_{t-1}|}{\Delta t}$
- Total number of errors – $\sum_{|e_t| > \varepsilon, |e_{t-1}| \le \varepsilon} 1$

C. Task

In each condition users performed a target-tracking task consisting of steering a car on a randomly moving road by using 1-DOF joystick. In the events of failure, i.e. when the car is outside the road, a sound is played back, which augments the visual with an audio feedback. Each session lasted 2 minutes. Performance data was logged on every frame.

D. Methodology

Twelve participants (9 male, 3 female, aged 18 to 48) took part in the study; all but one reported right dominant hand. We used a within-subjects design and conditions were randomized and tested in a sitting lab environment. The experiment consisted of a short introduction followed by a training session and the three conditions. The training session, devoted to familiarize users with the controls, lasted until a certain level of performance was achieved and usually took less than 10 minutes to complete. It was divided into four parts: in the first
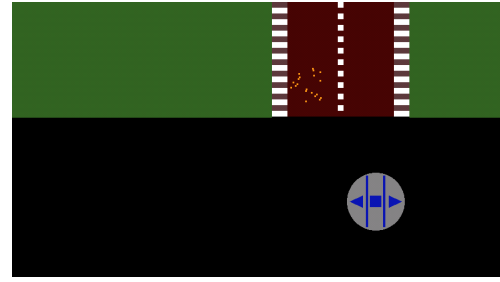

Fig. 5: Visual display (upper half) in Low empowerment (car location is a single point in a wider point cloud); touch-sensitive control area (gray sphere/lower half).

part users experienced the joystick controls and then practiced all three versions of the game. All conditions represented identical random sequences of a horizontally moving road which subjects had to track for 2 minutes. At the end of each condition they were asked to complete a NASA-TLX [2] and a free-form questionnaire, rating perceived workload and level of engagement respectively. Once all conditions were completed a questionnaire assessing user experience and preferences was also filled in. The experiment took about half an hour in total and participants were allowed to rest between conditions.

V. RESULTS

A. Performance

A non-parametric Friedman test showed a significant effect on level of empowerment per condition for the total number of errors (χ^2=18.77, df=2, p<0.0001), the total time off-target (χ^2=11.17, df=2, p<0.005), the average duration on-target (χ^2=18, df=2, p<0.0001), the perceived uncertainty (χ^2=19.02, df=2, p<0.0001), the perceived performance (χ^2=10.36, df=2, p<0.01) and the frustration (χ^2=13.74, df=2, p<0.001). Pair-wise Wilcoxon signed ranks tests showed that subjects performed significantly better in the High empowerment condition than in the Medium and Low empowerment
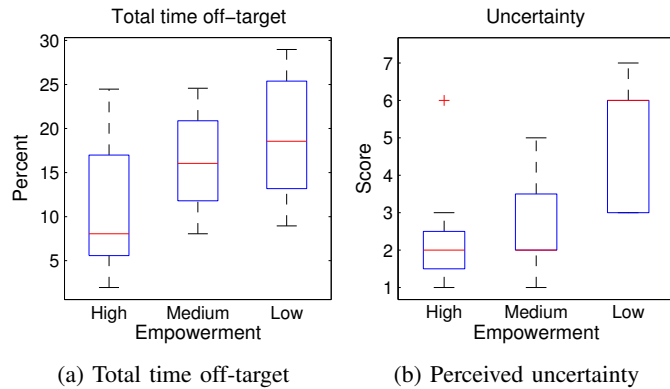

(a) Total time off-target

(b) Perceived uncertainty

Fig. 4: (a) Time spent outside the road. High empowerment is significantly lower than Medium (p=0.024 with Bonferroni correction) and Low ($p < 0.005$); (b) Subjective uncertainty about the exact location of the car at any given moment, measured on a 7-point Likert scale. Low empowerment is significantly higher than the other two ($p < 0.005$).
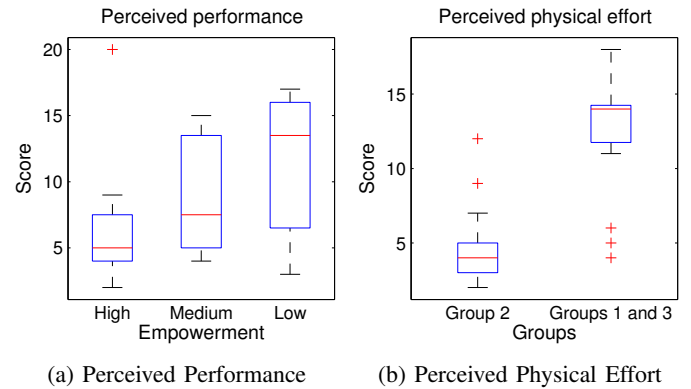

(a) Perceived Performance

(b) Perceived Physical Effort

Fig. 6: Subjective measures on a 20-point NASA-TLX scale: (a) within-subjects Perceived Performance – High is significantly better than Low (p=0.014 with Bonferroni correction); (b) between-groups Perceived Physical Effort – Group 2 is significantly lower than Groups 1 and 3 combined (all conditions combined) ($p < 0.00001$).
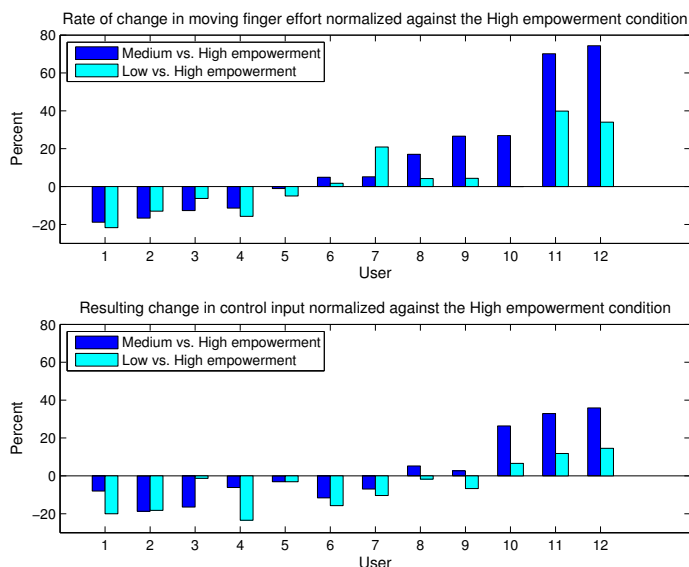
Fig. 7: Change in finger movement effort (upper plot) and resulting change in control input (lower plot) – users are sorted according to performance in Medium vs. High empowerment on the upper plot and matched on the lower one. Medium and Low empowerment conditions are normalized against High.

conditions – i.e. had lower total number of errors (p<0.001), higher average duration on-target (p<0.001), lower total time off-target (p=0.024 with Bonferroni correction) and (p<0.005) for Medium and Low empowerment condition respectively (Fig. 4a). Perceived uncertainty in the Low empowerment condition was significantly higher than in the High and Medium empowerment conditions (p<0.005) (Fig. 4b) and perceived performance in the High empowerment condition was significantly better than in the Low empowerment condition (p=0.014 with Bonferroni correction) (Fig. 6a), consistent with the measured performance. Our study did not provide significant evidence regarding the rest of the metrics, thus the respective data will not be presented here in detail.

B. Behaviour

Different users had different ways of coping with noise, reflected by changes in their activity levels driven by varying levels of disturbances, visible (Fig. 7) in the rate of change in finger movement effort and the resulting change in control input. Grouping users into Group 1 (1,2,3,4), Group 2 (5,6,7,8,9) and Group 3 (10,11,12), we see that Group 1 reacted to increased disturbances by decreasing the expended physical effort and actual control by nearly 20%, i.e. by adopting a more relaxed behavior, while Group 3 reacted by increasing them by 20% and more, i.e. by putting more effort into dealing with

TABLE II: Subjective user preferences

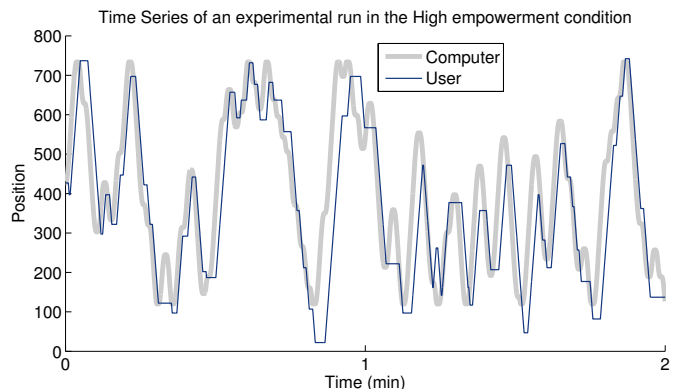| Condition (empowerment)	High	Medium	Low
Most preferred	9	3	0
Least preferred	1	4	7

Fig. 8: Time series of a High empowerment session – an example for a low expended effort (Total time off-target=24%, Perceived performance=20). The lag patterns of relaxed target-following behaviour are clearly visible around the peaks.

disturbances. This shows that Group 1 expended more effort the more empowered they were to control the environment. Group 2 expended lower levels of effort on average and had nearly constant levels in all conditions, however it showed an interesting trend as well, as the changes in finger movement effort and in resulting control input are not reciprocal. Interestingly, higher levels of finger movement effort resulted in lower levels of control input. This discrepancy can be attributed to the loss of empowerment induced by the higher level of disturbances. More precisely – subjects did not perceive accurately the effect of their actions in the Medium and Low empowerment conditions. Furthermore, in the subjective preferences (Table II) only users from Group 2 (all but one) disliked mostly the Medium empowerment condition, which had more erratic and stressful visual feedback. Group 2 had also lower scores in Perceived Physical Effort in all conditions and the difference between Group 2 and Groups 1 and 3 combined (all conditions combined) is significant (Fig. 6b), shown by pair-wise Wilcoxon rank sum test ($p < 0.00001$).
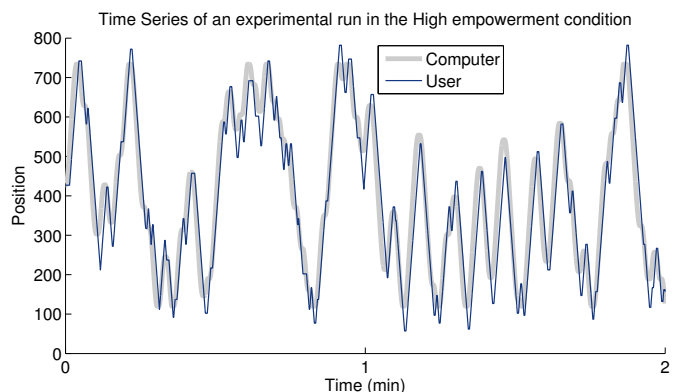

Fig. 9: Time series of a High empowerment session-an example for a high expended effort (Total time off-target=2%,Perceived performance=4). The lead patterns of eager target-anticipating behaviour are clearly visible at the turning points.

C. Examples

Most subjects developed a strategy of anticipating where the road was going to move next and applied that in all conditions. However the response time varied significantly across subjects – some were faster and others slower to react. Time series of two users show different patterns of lead and lag behaviour, the first one being more relaxed (Fig. 8) and the second one – more active (Fig. 9); their performance figures appear on the two extremes in Fig. 4a and 6a. Eager anticipation of more active users led to cases of oscillation at repeated turning points, while more relaxed ones showed smoother tracking behaviour.

VI. Discussion

Results show that performance in the Low and Medium empowerment conditions dropped significantly compared to the High empowerment condition, which was expected, however this was not linked to a consistent trend in control effort, since some users increased and others decreased their effort depending on their attitude. This shows the different ways users adapt their behaviour in the face of uncertainty. The analysis also revealed a particular group of subjects, who had kept their control effort quasi steady across conditions and who interestingly disliked the Medium empowerment condition most (and not the Low empowerment one), and furthermore had significantly lower perceived physical effort than the rest. Low and Medium empowerment conditions were significantly different in perceived uncertainty which is a reflection of the system design, however this did not translate proportionally into performance difference, as they were close both in measured and perceived performance. Higher uncertainty also increased frustration. This shows that in a very uncertain environment humans are able to adapt appropriately (i.e. increasingly rely on prior knowledge as suggested in [7]) and succeed in delivering a relatively good performance despite increased frustration. The lack of significant differences in performance between the Low and Medium empowerment conditions can also be attributed to the smaller gap in empowerment level than the one between the High and Medium empowerment conditions.

The relations of the means of total time off-target, perceived uncertainty, perceived performance and frustration vs. *empowerment* are strictly monotonic with expected trends, which demonstrate the overlap of *empowerment* with standard measures. However, the large variability in the data shows that further work is required to establish the exact regression curves. Loss of *empowerment* implicated interesting trends in activity levels, indicating another exciting area for future work. These results, along with the theoretical coherence of *empowerment* suggest its potential as part of a future toolset for understanding interactive systems.

Applying the *empowerment* measure, however, requires prior theoretical modelling, which, depending on the particular system, may become too costly. There is a trade-off between the accuracy of the theoretical models and the reliability of the *empowerment* measure – the more accurate the models, the more costly they are to create, but the more reliable the measure they imply.

Technology and design related usability issues affected our prototype system. Most participants lacked the tactile feedback from the control area covered by their finger, which increased the difficulty in controlling the discrete transitions at this specific resolution level. Few participants used most of the time the two extremes – left and right – and less so the middle, perhaps due to the narrow range. All subjects found it easy to learn how to use the system and found the audio cue beneficial.

VII. Conclusion

In this paper we presented an evaluation of how people perform in noisy environments and proposed a way of adapting the *empowerment* measure to quantify uncertainty in manual control tasks. We have shown that the relation of the new measure to the means of a collection of standard performance metrics is strictly monotonic, which suggests its potential in making predictions and giving theoretical bounds on empirical performance measures, based solely on properties of the environment. It is conceptually a new quantity, which integrates different types of noise in a single theoretical measure reflecting uncertainty from the agent's point of view. A benefit of the approach is in providing an analytical tool for performance tuning, before resorting to costly empirical studies. A drawback is that it requires prior theoretical modelling – the more accurate the models, the more costly they are to create, but the more reliable the measure they imply. Further work is required to investigate the properties of the proposed measure and to validate the approach in other areas of HCI. The results presented here are important to raise the awareness of the community about the potential empowerment has in providing better theoretical foundations for the science of HCI.

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