Worksheet 1 (Tutorial)

This worksheet contains exercises on some of the material covered in lecture 2. You should attempt the exercises before your tutorial. During the tutorial, your tutor will work through any exercises which have caused problems. These exercises are not assessed.

Conversion from Binary to Decimal

Convert the following unsigned binary numbers into decimal.

(a) 110100 (b) 01011011 (c) 10010110 (d) 111111 (e) 1111110

Conversion from Decimal to Binary

Convert the following decimal numbers into binary. How many bits are needed in each case?

(a) 14 (b) 127 (c) 249 (d) 73 (e) 257

Addition in Binary

Work out the following sums in unsigned binary. In each case, check your answer by converting the numbers and the result into decimal.

(a) 11011 + 1101 (b) 11010101 + 00111110 (c) 1111 + 1 (d) 111111 + 1
(e) 10101010 + 01010101

2s Complement

Work out the 2s complement binary representation of the following decimal numbers, using 8 bits.

(a) 97 (b) -127 (c) -29 (d) -76 (e) -2

Convert the following 2s complement binary numbers to decimal.

(a) 10000000 (b) 11111100 (c) 10101010 (d) 00001101 (e) 11010111

Subtraction

Work out the following subtractions in 2s complement binary (do this by negating the second number and then adding). Use an 8 bit representation. In each case, check your answer by converting the numbers and the result into decimal.

(a) 00011011 - 00001101 (b) 11010101 - 00111110 (c) 00001111 - 00000001
(d) 00111111 - 00000001 (e) 10101010 - 01010101

Hexadecimal

Convert the following hexadecimal numbers into binary and decimal.

(a) 3A (b) FF (c) A5 (d) 32 (e) BD

Convert the following decimal numbers into hexadecimal (an easy way to do this is to convert into binary first).

(a) 254 (b) 256 (c) 64 (d) 181 (e) 99

Text Representation

The ASCII character set represents each character by a 7 bit binary number, which can also be converted into a 2 digit hexadecimal number. The ASCII codes for the capital letters A–Z are 65–90 (in decimal). A complete table of the ASCII codes (in decimal) can be found on page 64 of *Computer Science Illuminated* or in many other books.

Work out the ASCII representation of your name, in hexadecimal. For example: SIMON is 83,73,77,79,78 in decimal and 53,49,4D,4F,4E in hex.

Supplementary

Fractional numbers can be represented in binary in a similar way to decimal: there is a "binary point", and the columns to the right of the point have value $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and so on.

For example, 11.01_2 is $3 + \frac{1}{4}$, which is 3.25_{10}.

Exercise: convert the following binary fractions to decimal.

(a) 0.1 (b) 0.11 (c) 0.101 (d) 0.1001 (e) 0.1111

Exercise: work out a systematic way of converting decimal fractions to binary.

Some fractions lead to recurring decimal representations: for example, $\frac{1}{3} = 0.3333\cdots$, sometimes written as $0.3^\bullet$. The same is true in binary. For example, here is a recurring binary fraction:

$$0.101010\cdots$$

which could be written as $0.(10)^\bullet$. To work out the value of this recurring fraction, x say, notice that $4x = 10.(10)^\bullet$ (because multiplying by 4 in binary means shifting the binary point two places to the right), and so

$$\begin{aligned} 4x &= x + 2 \\ 3x &= 2 \end{aligned}$$

and therefore $x = \frac{2}{3}$.

As it happens, $\frac{2}{3}$ also has a recurring decimal representation, but this is not always the case.

Exercise Can you find a non-recurring decimal which is recurring in binary?

Exercise Can you see why every non-recurring binary fraction is also non-recurring in decimal?