Worksheet 1 (Tutorial) Solutions

Conversion from Binary to Decimal

Convert the following binary numbers into decimal.

(a) 110100 (b) 01011011 (c) 10010110 (d) 111111 (e) 1111110

Solutions:

(a) 52 (b) 91 (c) 150 (d) 63 (e) 126

Conversion from Decimal to Binary

Convert the following decimal numbers into binary. How many bits are needed in each case?

(a) 14 (b) 127 (c) 249 (d) 73 (e) 257

Solutions:

(a) 1110 (b) 1111111 (c) 11111001 (d) 1001001 (e) 100000001

Two systematic conversion techniques have been covered in lectures.

Addition in Binary

Work out the following sums in binary. In each case, check your answer by converting the numbers and the result into decimal.

(a) 11011 + 1101 (b) 11010101 + 00111110 (c) 1111 + 1 (d) 111111 + 1
(e) 10101010 + 01010101

Solutions:

(a) 101000 (b) 100010011 (c) 10000 (d) 1000000 (e) 11111111

2s Complement

Work out the 2s complement binary representation of the following decimal numbers, using 8 bits.

(a) 97 (b) -127 (c) -29 (d) -76 (e) -2

Solutions:

For negative numbers: convert the positive number to binary, invert, add 1.

For positive numbers: just convert to binary.

(a) 01100001 (b) 10000001 (c) 11100011 (d) 10110100 (e) 11111110

Convert the following 2s complement binary numbers to decimal.

(a) 10000000 (b) 11111100 (c) 10101010 (d) 00001101 (e) 11010111

Solutions:

For negative numbers (with a leading 1): negate by inverting and adding 1, then convert to decimal, then negate.

For positive numbers (with a leading 0): just convert to decimal.

(a) -128 (b) -4 (c) -86 (d) 13 (e) -41

Subtraction

Work out the following subtractions in binary (do this by negating the second number and then adding). Use an 8 bit representation. In each case, check your answer by converting the numbers and the result into decimal.

(a) 00011011 - 00001101 (b) 11010101 - 00111110 (c) 00001111 - 00000001
(d) 00111111 - 00000001 (e) 10101010 - 01010101

Solutions:

(a) 00001110 (b) 10010111 (c) 00001110 (d) 00111110 (e) 01010101

Hexadecimal

Convert the following hexadecimal numbers into binary and decimal.

(a) 3A (b) FF (c) A5 (d) 32 (e) BD

Solutions:

In binary (each hex digit becomes 4 bits):

(a) 00111010 (b) 11111111 (c) 10100101 (d) 00110010 (e) 10111110

In decimal (16 $\times$ first digit + second digit):

(a) 58 (b) 255 (c) 165 (d) 50 (e) 189

Convert the following decimal numbers into hexadecimal (an easy way to do this is to convert into binary first).

(a) 254 (b) 256 (c) 64 (d) 181 (e) 99

Solutions:

Convert to binary, then convert each group of 4 bits to a hex digit.

(a) 11111110, FE (b) 000100000000, 100 (c) 01000000, 40 (d) 10110101, B5
(e) 01100011, 63

Supplementary

Fractional numbers can be represented in binary in a similar way to decimal: there is a "binary point", and the columns to the right of the point have value $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and so on.

For example, 11.01_2 is $3 + \frac{1}{4}$, which is 3.25_{10}.

Exercise: convert the following binary fractions to decimal.

(a) 0.1 (b) 0.11 (c) 0.101 (d) 0.1001 (e) 0.1111

Solutions:

(a) $\frac{1}{2} = 0.5$
(b) $\frac{1}{2} + \frac{1}{4} = 0.5 + 0.25 = 0.75$
(c) 0.101 $\frac{1}{2} + \frac{1}{8} = 0.5 + 0.125 = 0.625$
(d) $\frac{1}{2} + \frac{1}{16} = 0.5 + 0.0625 = 0.5625$
(e) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 0.5 + 0.25 + 0.125 + 0.0625 = 0.9375$

Exercise: work out a systematic way of converting decimal fractions to binary.

Solution: (there may be others)

Multiply by 2. If the result is at least 1, the next digit is 1, otherwise it is 0. If the result is at least 1, subtract 1. Repeat until a result of 0 is obtained (this might never happen).

The digits are generated from left to right.

Recurring decimals

Some fractions lead to recurring decimal representations: for example, $\frac{1}{3} = 0.3333\cdots$, sometimes written as $0.3^\bullet$. The same is true in binary. For example, here is a recurring binary fraction:

$$0.101010\cdots$$

which could be written as $0.(10)^\bullet$. To work out the value of this recurring fraction, x say, notice that $4x = 10.(10)^\bullet$ (because multiplying by 4 in binary means shifting the binary point two places to the right), and so

$$\begin{aligned} 4x &= x + 2 \\ 3x &= 2 \end{aligned}$$

and therefore $x = \frac{2}{3}$.

As it happens, $\frac{2}{3}$ also has a recurring decimal representation, but this is not always the case.

Exercise Can you find a non-recurring decimal which is recurring in binary?

Solution:

Perhaps the simplest example is $\frac{1}{5}$, which is 0.2 in decimal but $0.(0011)^\bullet$ in binary. Using similar working to the above, if $x = 0.(0011)^\bullet$ we have

$$\begin{aligned} 16x &= x + 3 \\ 15x &= 3 \end{aligned}$$

and therefore $x = \frac{1}{5}$.

Exercise Can you see why every non-recurring binary fraction is also non-recurring in decimal?

Solution:

Because every binary column value ($\frac{1}{2}$, $\frac{1}{4}$ etc.) is a non-recurring decimal. So adding a finite number of them together (which is what a non-recurring binary fraction means) results in a non-recurring decimal.