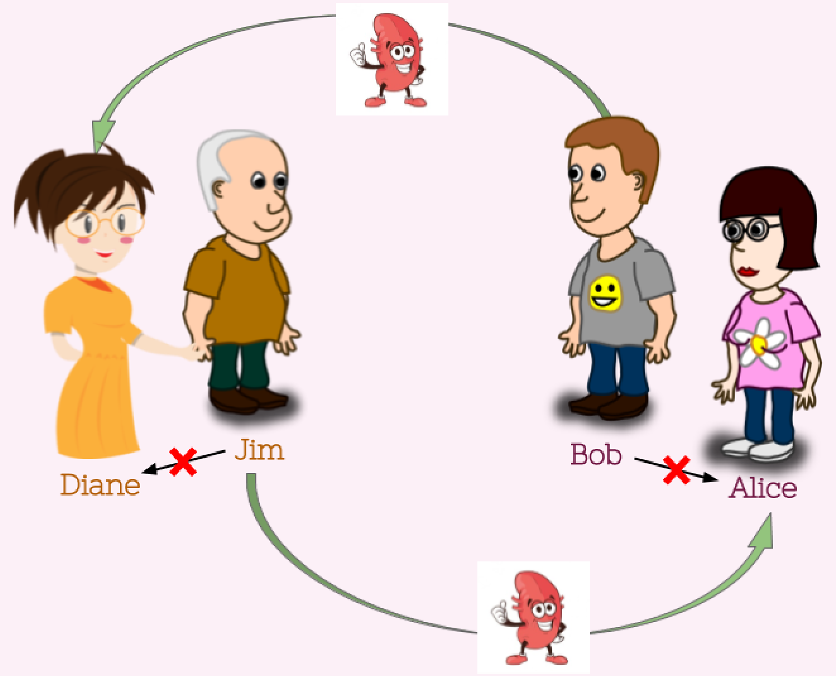
Algorithmic Results on the Student-Project Allocation Problem

Sofiat Olaosebikan* and David Manlove, School of Computing Science, University of Glasgow

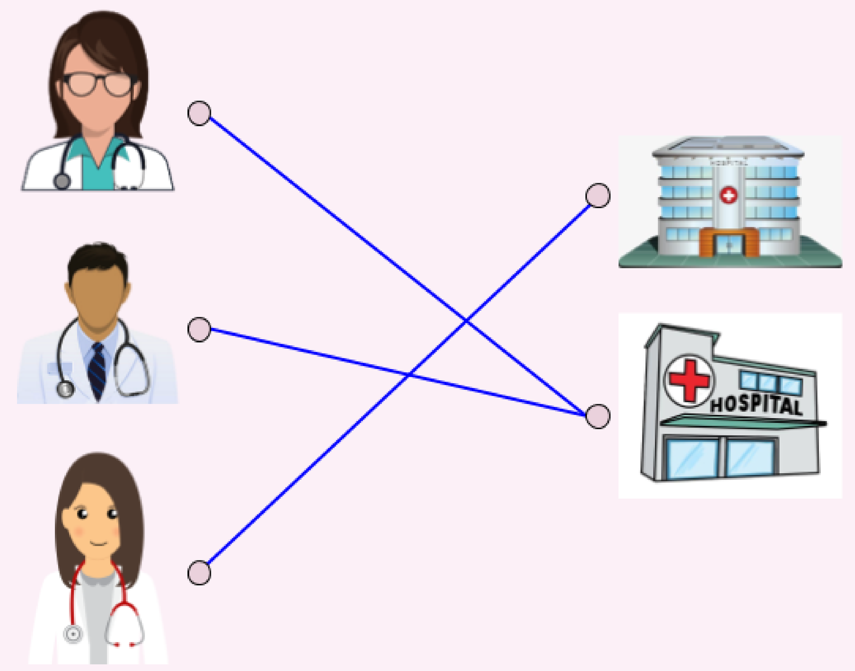
University of Glasgow

Background

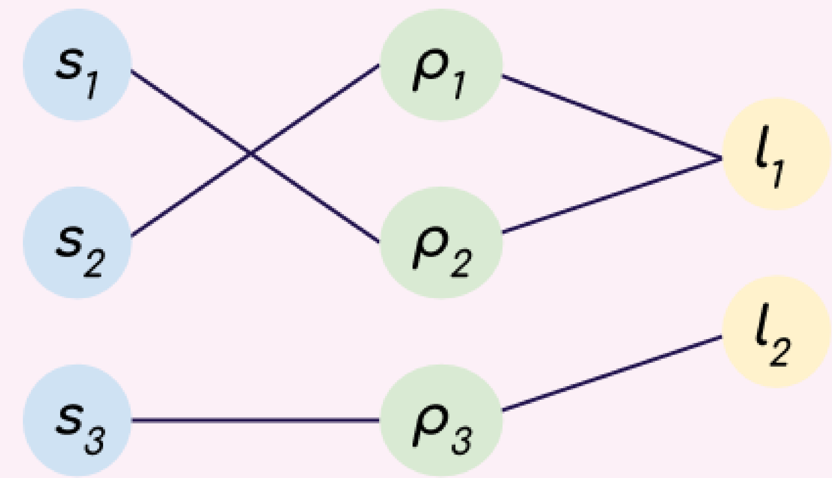
Matching problems arise when we need to find an optimal allocation between sets of agents. A typical trend in all of the applications is that the agents involved have preferences over the possible outcome and each agents typically have a specified *capacity*, which is the maximum number of agents they can accommodate. The goal is to find a *matching*, i.e., an allocation of these agents to one another that takes their preferences and capacities into consideration. When all of the agents involved have preferences, we care about the *stability* of the matching. A matching is stable if there is no pair of agents who would rather be assigned together than accept their current assignment. In practice, such agents could potentially undermine the integrity of the matching.



(a) Since 2007, NHS Blood and Transplant have run an algorithm that solves the Kidney exchange problem.



(b) The National Resident Matching Program in the United States employs a matching algorithm to allocate junior doctors to hospital posts.



(c) The School of Computing Science, University of Glasgow uses an algorithm to allocate students to dissertation projects.

Problem 1

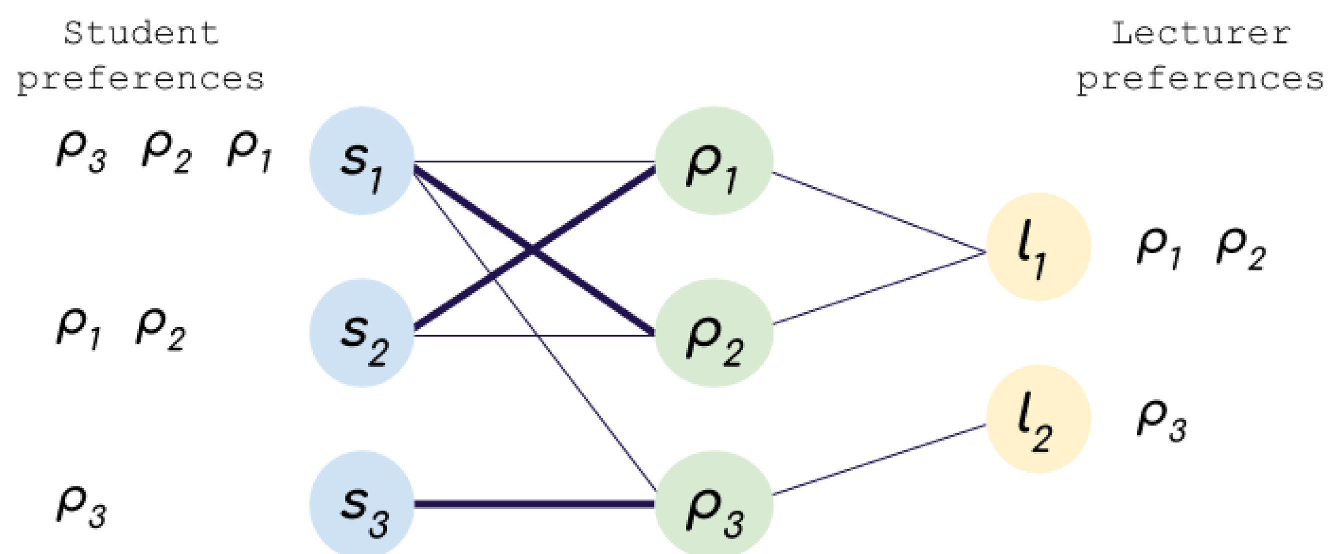


Figure 1: Each project has capacity 1. Lecturers l_1 and l_2 have capacities 2 and 1 respectively. Student s_1 prefers p_3 to p_2, and so on.

Model problem: Student-Project Allocation with students and lecturers having preferences over projects (SPA-P).

What we seek: A stable matching that assigns as many students to projects as possible (MAX-SPA-P).

Existing results: MAX-SPA-P is NP-hard. There are two approximation algorithms guaranteed to produce stable matchings that are at least $\frac{1}{2}$ and $\frac{2}{3}$ the size of the optimal solution.

Our first contribution: An Integer Programming model to enable MAX-SPA-P to be solved to optimality [2]. In Fig. 1, the maximum stable matching is: $s_1 - p_2$, $s_2 - p_1$ and $s_3 - p_3$.

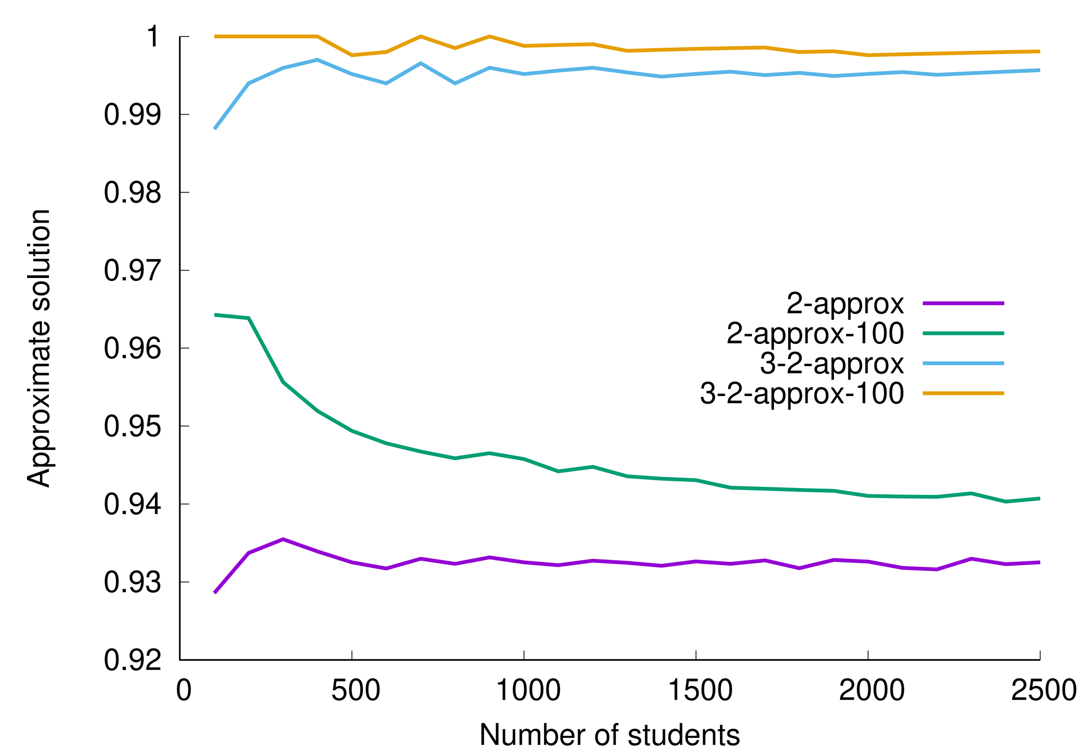


Figure 2: An empirical analysis that compares the approximation algorithms and the IP model based on randomly generated SPA-P instances. The solution produced by the $\frac{3}{2}$-approximation algorithm is extremely close to optimal.

Problem 2

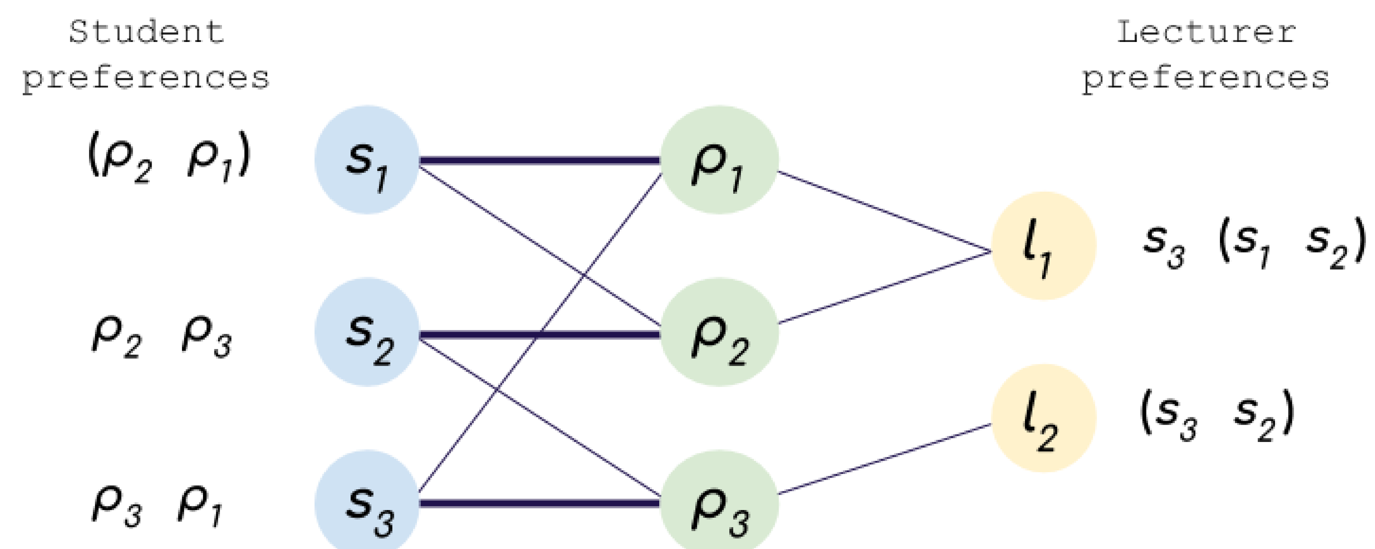


Figure 3: Each project has capacity 1. Lecturers l_1 and l_2 have capacities 2 and 1 respectively. Student s_1 is indifferent between p_2 and p_1.

Model problem: Student-Project Allocation with students preferences over projects, lecturers preferences over students, and with ties (SPA-ST).

What we seek: Three stability concepts are possible: (i) weak stability, (ii) strong stability and (iii) super-stability.

Existing results: Under weak stability, the problem of finding a stable matching that assigns as many students to projects as possible is NP-hard. A $\frac{3}{2}$-approximation algorithm is described in [1].

Our second contribution: A polynomial-time algorithm to find a strongly stable matching or report that no such matching exists [4]. In Fig. 3, the strongly stable matching is: $s_1 - p_1$, $s_2 - p_2$ and $s_3 - p_3$.

Our third contribution: A polynomial-time algorithm to find a super-stable matching or report that no such matching exists [3]. In Fig. 4, the super-stable matching is: $s_1 - p_1$ and $s_3 - p_3$.

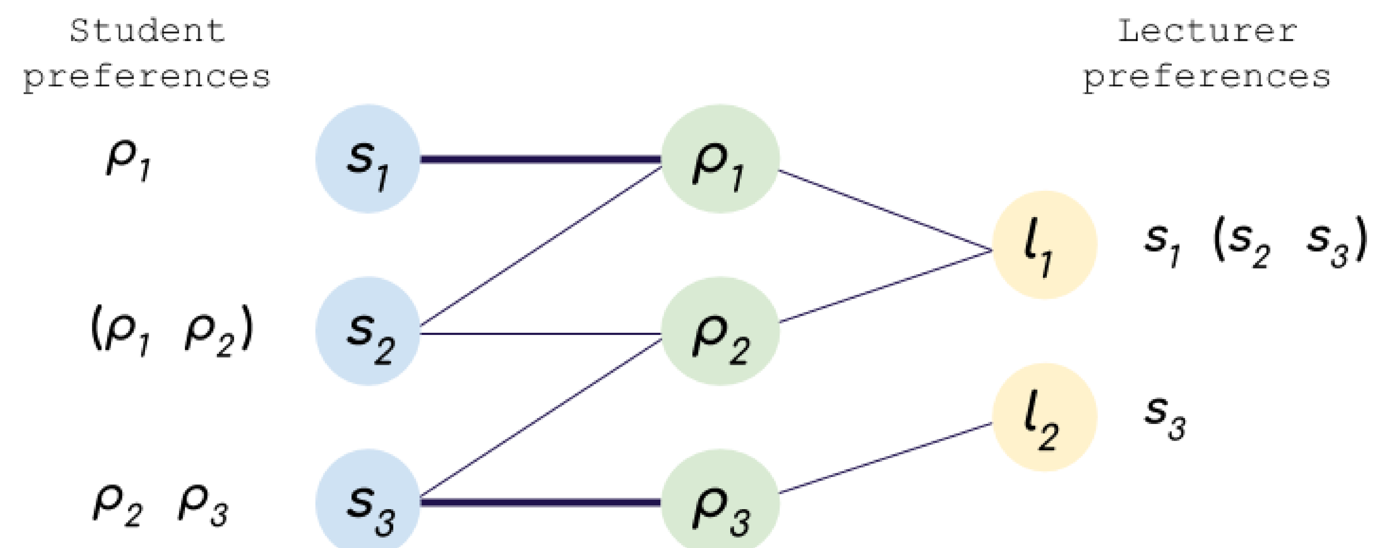


Figure 4: Each project and lecturer has capacity 1.

References

[1] F. Cooper and D.F. Manlove. A 3/2-approximation algorithm for the Student-Project Allocation problem. In *Proceedings of SEA 2018*, vol. 103 of *LIPIcs*, pgs 8:1–8:13.

[2] D.F. Manlove, D. Milne, and S. Olaosebikan. An Integer Programming Approach to SPA-P. In *Proceedings of ISCO '18*, vol 10856 of *LNCS*, pgs 313 – 325. Springer, 2018.

[3] S. Olaosebikan and D. Manlove. Super-Stability in the Student-Project Allocation Problem with Ties. In *Proceedings of COCOA '18*, vol 11346 of *LNCS*, pgs 357 – 371. Springer, 2018.

[4] S. Olaosebikan and D. Manlove. An Algorithm for Strong Stability in the Student-Project Allocation Problem with Ties. Accepted for *MATCH-UP '19*.

*Supported by a College of Science and Engineering Scholarship, University of Glasgow.

s.olaosebikan.1@research.gla.ac.uk | @soolaosebikan | www.dcs.gla.ac.uk/~sofiat